# THE ALTERNATING CURRENT TRANSFORMER

~~1898~~

BY

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# PREFACE

The following pages originated from a course of University lectures. It is hoped that the book will be of use to the engineer and general reader. Some knowledge of elementary alternating currents is presupposed.

I have found that the student who grasps the theory of the transformer has little difficulty with the more complicated actions and reactions taking place in the induction motor, and is also helped in his study of synchronous apparatus.

Chapters I and II are merely introductory. Chapters III to V, carefully studied, should give the reader a thorough knowledge of the transformer. The tests in Chapter VI should be carried out by the student on a few commercial types. The method of design in Chapter VII is offered as a probable improvement over that usually given in text books.

I wish to thank the manufacturers for the loan of cuts used in Chapter X.

Thanks are also due the publishers who have maintained their reputation for sparing no effort or expense to make the book mechanically as perfect as possible. I am indebted to Mr. A. S. Kalenborn for reading proofs.

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# NOTATION.

$i$=instantaneous value of the current,
$I$=maximum value of the current,
$n$=instantaneous value of the magnetic flux,
$N$=maximum value of the magnetic flux,
$h$=instantaneous value of the strength of field,
$H$=maximum value of the strength of field,
$S$=number of turns,
$a$=ratio of transformation$=\frac{S_1}{S_2}$,
$L$=coefficient of self-induction,
$M$=coefficient of mutual induction,
$l$=length of magnetic circuit,
$A$=cross section of iron under induction,
$e$=instantaneous value of e.m.f.,
$E$=maximum value of e.m.f. or effective e.m.f.,
$\omega$=two $\pi$ times the frequency,
$f$=frequency,
$R_1$=primary internal resistance,
$r_2$=secondary internal resistance,
$R_2$=resistance of load,
$E_2$=e.m.f. applied to load,
$E_1$=primary induced e.m.f.,
$\frac{E_1}{a}$=secondary induct e.m.f.,

NOTE.—Subscripts $_1$ and $_2$ refer to primary and secondary coil respectively.

# THE

# ALTERNATING CURRENT TRANSFORMER

## CHAPTER I. 1898

### ELEMENTARY PRINCIPLES.

**1.** The alternating current transformer consists of a magnetic circuit interlinked with two or more electric circuits. The simplest arrangement is shown in Fig. 1.

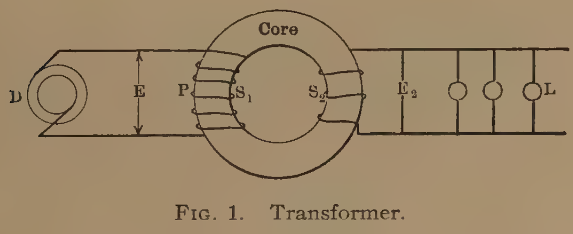


FIG. 1. Transformer.

The primary coil, $P$, consumes power from the generator; in the secondary coil, $S$, electric pressure is developed, and when the external circuit is closed by some external translating device, $L$, power appears in the secondary circuit. Power is, therefore, transferred through space from the primary to the

secondary circuit without relative motion between the two coils. This interchange of power takes place through the medium of the magnetic field which threads the primary and secondary coils.

The electric pressure developed in the secondary is, approximately,

$$E_2 = \frac{ES_2}{S_1} = \frac{E}{a};$$

that is, the primary pressure is reduced in the ratio of the number of turns. The transformer is used, therefore, because it is a convenient method of changing the electric pressure. It finds its greatest use in transforming power from high to low pressure at a point near the consumer of energy.

Before taking up the effects of the current in one coil upon the other, it will be well to review briefly the fundamental principles of the magnetic circuit and the laws of magnetic induction.

**2. Magnetic Induction.**—Consider a current flowing in a closed circuit of one turn of wire. A magnetic field is produced by the current, consisting of lines of force threading the circuit and forming closed curves. This magnetic field or flux is proportional to the current in the wire, any change in the current producing a corresponding change in the total flux, $N$. If there are $S$ turns of wire, the lines of force pass through each turn, and there are, therefore, $SN$ lines of force threading a circuit of $S$ turns.

**3. Magnetomotive Force or Difference of Magnetic Potential.**—Preliminary proposition: The work, $W$, done on a circuit in maintaining a current, $I$, in that circuit constant while the magnetic flux through the circuit changes by an amount, $N$, is

$$W = IN. \tag{1}$$

Proof: Suppose we have a wire, of length $l$, carrying a current $I$, at right angles to a magnetic field

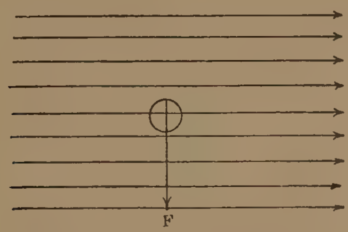

FIG 2. Force on Wire.

of strength $H$, as shown in Fig. 2. The force acting on the wire at right angles to the field and the wire is

$$F = IlH.$$

Now, move the wire against this force through a distance $x$; the work done is

$$W = IlHx = IN,$$

since $l\,x\,H = N$ the flux change.

The work done on unit pole in moving it once around the closed path, as indicated by the dotted line in Fig. 3, against the magnetic forces of the system is a measure of the power of that system to magnetize, or, in other words, is a measure of its magnetomotive

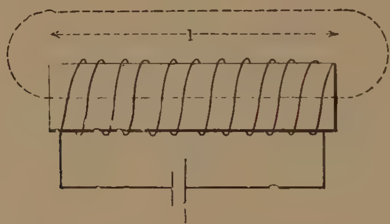

FIG. 3. Straight Solenoid.

force. The magnetomotive force between two points in a magnetic field is measured by the work done in moving a unit magnetic pole from one point to the other. Hence,

Magnetomotive force = fall of magnetic potential.

Example: Let us find the m.m.f. of the system shown in Fig. 3. If the solenoid is uniformly wound and the layers are near together, we may consider a current of magnitude $SI$ flowing in a single sheet around the coil, instead of the current in each wire. Move the unit pole around the path indicated by the dotted line. Each of the $4\pi$ lines of force radiating

from unit pole* must be cut by the current $SI$. The work done, therefore, according to equation (1), is

$$W = 4\pi\, SI, \qquad (2)$$

and this is the m.m.f. of the system. The two ends of the solenoid may be brought together to form a

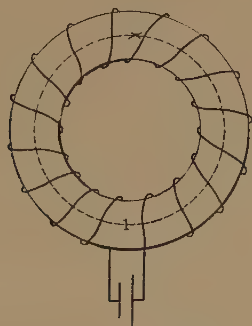

FIG. 4. Ring Solenoid.

circular coil as in Fig. 4; the m.m.f. of the system remains the same.

**4. Strength of Field in Solenoid.**—If, in Fig. 4, the strength of field in the solenoidal ring be taken as $H$, the work done in moving unit pole once around the circuit is $Hl$, and, further, if the lines of force due to one turn be supposed to pass through all the

*By definition $4\pi$ lines of force radiate from a unit pole.

other turns on the coil, this work will be equal to the total m.m.f. of the coil. That is,

$$Hl = 4\pi\ SI, \text{ or} \tag{3a}$$
$$H = 4\pi\ SI/l. \tag{3b}$$

If the material forming the ring on which the wire is wound has a permeability $\mu$ and a cross section $A$, the total flux is

$$N = A\ \mu\ H = 4\ \pi\ SIA\ \mu\ /l$$
$$= (4\ \pi\ SI)/(l/A\mu). \tag{4}$$

The denominator in the right-hand member of equation (4) is defined as the magnetic reluctance of the circuit. In words the above equation is

$$\text{Magnetic flux} = \frac{\text{Magnetomotive force}}{\text{Magnetic reluctance}}.$$

5. **Induced E.M.F.**—If the magnetic induction through any circuit changes, *due to any cause whatever*, an e.m.f. is developed in the circuit proportional to the rate of change of magnetic induction (Faraday's Law). That is

$$e = -dn/dt.^* \tag{5a}$$

If $n$ is the magnetic flux passing through each turn and there are $S$ turns

$$e = -S\ dn/dt. \tag{5b}$$

---

*The negative sign is used because a positive flux change gives a negative e. m. f.

The strength of field at any point in a magnetic circuit of a (stationary) current is proportional to the current; that is

$$h \propto Si \qquad \text{(Experimental Law).}$$

If the permeability of the medium is constant,* the number of lines of force threading the circuit in which the current is flowing (or any closed circuit in the neighborhood) will be proportional to $i$; that is

$$n \propto h \propto Si, \text{ or } n = k\,Si. \qquad (6)$$

We have, then, from (5$b$) and (6)

$$e = -S\,dn/dt = -kS^2\,di/dt, \text{ or}$$

$$e = \frac{-Sdn}{dt} = \frac{-Ldi}{dt} \qquad (7a)$$

$$\therefore L = \frac{SN}{I} \qquad (7b)$$

$L$ is defined as the coefficient of self-induction of the circuit. If the induced e.m.f. equals one C.G.S. unit e.m.f. when the current varies at the rate of one C.G.S. unit per second, that is $di/dt = 1$, the circuit is said to have one C.G.S. unit of self-induction. If in equation (7$a$), $e$ is in volts, $i$ in amperes and $t$ in seconds, the unit in which $L$ is expressed is the henry.

One henry $= 10^9$ C.G.S. units of self-induction.

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* On account of the low magnetic density in transformers, the permeability may be considered as constant without appreciable error.

From (7*b*) another definition for $L$ may be given: The self-induction of any circuit is measured by the number of lines of force which cut the circuit when the current increases or decreases by one unit.

**6. Coefficient of Self-induction Calculated from the Constants of the Coil.**—We have (Fig. 4),

$$e = -L\, di/dt = -S\, dn/dt$$

But

$$n = 4\pi\, SIA\mu/l, \text{ giving}$$
$$dn/dt = (4\pi\, SA\mu/l)\, di/dt, \text{ or}$$
$$L\, di/dt = (4\pi\, S^2\, A\mu/l)\, di/dt, \text{ giving}$$
$$L = 4\pi\, S^2\, A\mu/l. \qquad (8a)$$

If the core is circular in cross section, of radius $r$, we have

$$A = \pi r^2;$$

and if $C$ is the total length of wire wound on the core we have

$$C = 2\pi\, rS \qquad \text{(nearly)}.$$

Substituting in $L$, we obtain

$$L = \mu C^2/l. \qquad (8b)$$

To reduce to practical units divide by $10^9$.

This formula shows that the self-induction of a solenoid is practically independent of the radius of the iron core and depends only on the permeability, length of iron core, and on the length of wire. The formula will enable us to estimate very rapidly the

self-induction of coils when the length of wire is known.

For a coil with length of iron core $l_i$, and air-gap $l_a$, we have,

$$L = C^2/(l_i/\mu + l_a) \qquad (8c)$$

$l_i/\mu$ is usually small in comparison with $l_a$; neglecting the latter term we have

$$L = C^2/l_a; \qquad (8d)$$

that is, the self-induction of a coil of this kind is equal to the square of the length of wire divided by the length of the air-gap. The length of wire and air-gap must be expressed in centimetres.

7. **Mutual Induction.**—If we have, in the neighborhood of a circuit in which the current is varying, a second closed circuit the changing flux in the first coil will, in general, induce a current in the second. The e.m.f. induced will be given by the equation,

$$e_2 = - S_2 \, (dn_2/dt).$$

Here $dn_2$ is the number of lines of force which cut the second circuit in the time $dt$. Now, in whatever position this coil be placed relative to the first coil, the number of lines of force threading it will be proportional to the current in the first coil; that is

$$n_2 \propto S_1 i_1, \text{ or}$$
$$n_2 = kSi_1.$$

Combining the above equations we obtain

$$e_2 = -S_2\, dn_2/dt = -kS_1\, S_2\, (di_1/dt), \text{ or}$$
$$e_2 = -M\, (di_1/dt). \qquad (9)$$

$M$ is defined as the mutual induction between the two circuits. $M$ will be expressed in C.G.S. or practical units according as $e$ and $i$ are expressed in C.G.S. or practical units.

**8. Mutual Induction Calculated from the Constants of the Coils.**—Let us calculate the coefficient of mutual induction between two coils wound on the same core. In Fig. 1, consider the secondary carrying no current, that is, all the lamps turned off, and we have

$$e_2 = -M\, di_1/dt = -S_2\, dn_2/dt,$$
from $(4)/n_2 = 4\pi\, S_1\, i_1\, A\mu/l$ giving
$$dn_2/dt = (4\pi\, S_1\, A\mu/l)\, di_1/dt$$
$$\therefore\ M di_1/dt = (4\pi\, A\mu\, S_1\, S_2/l)\, di_1/dt, \text{ or}$$
$$M = 4\pi\, A\mu\, S_1\, S_2/l.$$

For a circular coil, this becomes

$$M = \mu C_1\, C_2/l. \qquad (10)$$

$C_1$ and $C_2$ being the lengths of wire on the two coils.

This is the mutual induction between the coils only on the assumption that all the lines of force thread both circuits, that is, when there is no

magnetic leakage. (We shall see later the effect of magnetic leakage.)

The self-induction of the secondary will be

$$L_2 = \mu C_2^2/l, \text{ and we see that}$$
$$M^2 = L_1 L_2, \text{ and} \tag{11}$$
$$L_1/L_2 = S_1^2/S_2^2. \tag{12}$$

**9. Induced E.M.F. in Secondary Coil (Secondary on Open Circuit).**—For the present we will assume a sine wave of e.m.f. applied to the primary, the resulting primary current being also assumed a sine wave. (We will take up the study of a distorted e.m.f. and current later.) With the secondary on open circuit, we have a simple series circuit containing resistance and self-induction. The primary current is

$$i_1 = \frac{E_1 \sin [\omega t - \tan^{-1} L_1 \omega/R_1]}{\sqrt{[R_1{}^2 + (L_1 \omega)^2]}}, \text{ or}$$
$$i_1 = I_1 \sin (\omega t - \tan^{-1} L_1 \omega/R_1).$$

The instantaneous value of the magnetic flux will, therefore, be

$$n_1 = \frac{4\pi S_1 A\mu I_1 \sin [\omega t - \tan^{-1} L_1 \omega/R_1]}{l}$$

The maximum value of $n$ is

$$N = (4\pi S_1 A\mu I_1)/l.$$

The curve for $n$ is given in Fig. 5.

Suppose this magnetic flux to vary through one cycle per second. Consider the number of times the lines of force cut the wires of the secondary circuit during one cycle. In building the lines up from zero to maximum value $N$, that is, in going from $o$ to $A$ on the curve, Fig. 5, all the lines of force cut the secondary turns. In going from $o$ to $A$, then, $S_2 N$ lines have cut the secondary. In going from $A$ to $B$, this number of lines must be with-

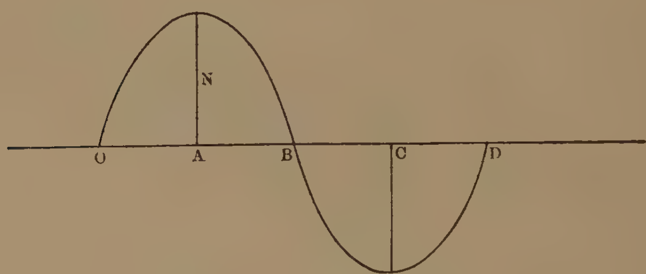



FIG. 5. Sine Wave of E. M. F. or Magnetic Flux.

drawn from the coil, and, therefore, again cut the secondary; similarly in going from $B$ to $C$ and from $C$ to $D$. Therefore, the number of lines of force which cut the secondary circuit in one cycle is $4S_2 N$. Since this number cut the secondary in one second, the average e.m.f. induced will be, by definition,

$$\text{Av. } E_2 = 4S_2 N/10^8 \text{ volts,}$$

and if the flux changes at a frequency of $f$ cycles per second

$$\text{Av. } E_2 = 4 S_2\, Nf/10^8, \text{ volts.} \qquad (13)$$

But we have for a sine wave

$$\begin{aligned} \text{Av. } E/\text{Max. } E &= 2/\pi; \\ \therefore \text{Max. } E_2 &= 2\pi\, S_2\, Nf/10^8. \end{aligned}$$

We also have

$$\begin{aligned} \text{Max. } E/\text{Vir. } E &= \sqrt{2}, \\ \therefore \text{Vir. } E_2 &= \sqrt{2}\pi\, S_2\, Nf/10^8. \qquad (14) \end{aligned}$$

(The virtual $E$ is the effective value or the square-root of the mean square.)

Equation (14) is of great importance and should be thoroughly understood.

$$\begin{aligned} \text{Since} \qquad \omega &= 2\pi f \qquad\qquad \text{and} \\ N &= 4\pi\, A\mu I_1/l, \\ \text{Max. } E_2 &= 4\pi\, A\mu S_1\, S_2\, \omega\, I_1/\,(l \times 10^8) \\ &= \frac{M\,\omega\, I_1}{10^8}. \qquad (15) \end{aligned}$$

**10. Primary Induced E. M. F.**—The changing magnetic flux which induces an e.m.f. in the secondary coil will also induce a "back" or counter e.m.f. in the primary. The back e.m.f. on the primary may be obtained from the secondary induced e.m.f. by substituting $S_1$ for $S_2$. This gives

$$\text{Max. } E_1 = 2\pi\, S_1\, Nf/10^8 = L\,\omega\, I_1/10^8. \qquad (16)$$

From equations (15) and (16) we have,

$$E_1/E_2 = S_1/S_2.$$

The above equations have been developed on the assumption that the secondary is carrying no load. Assuming no losses, this shows that if we have an e.m.f. $E_1$ applied to one coil of a transformer the e.m.f. set up in the other will be changed in the ratio of turns.

## CHAPTER II.

### SIMPLE TRANSFORMER DIAGRAM.

(The method given in this chapter is sometimes used in text books. I have found that the study of the transformer from this point of view always produces confusion in the mind of the student. The theory as given in Chapters III and IV is now considered the most straightforward.)

**11.** Under load, the actions and reactions of primary and secondary currents become somewhat complex. If a simple harmonic e.m.f. be applied to the primary (secondary loaded), we will have the following taking place:

(*a.*) A simple harmonic current is produced in the primary under an effective e.m.f. $I_1 R_1$, having the instantaneous value

$$i_1 = I_1 \sin \omega t.$$

(The phase of $i_1$ is here assumed as zero.)

(*b.*) A counter e.m.f. of self-induction, having a maximum value $L_1 \omega I_1$, is set up in the primary at right angles to $I_1 R_1$.

$$\text{Back } e_{1p} = -L_1 \frac{di_1}{dt} = -L_1 \omega I_1 \cos \omega t$$
$$= L_1 \omega I_1 \sin (\omega t - \pi/2).$$

(*c.*) An e.m.f., having a maximum value $M\omega I_1$, is induced in the secondary at right angles to $I_1$:

$$e_2 = - M\, di_1/dt = M\omega I_1 \sin (\omega t - \pi/2).$$

This has the same phase as the back e.m.f. on the primary.

(*d.*) A current $I_2$ flows in the secondary under an effective e.m.f. $I_2\, R_2$, the phase being behind the e.m.f. by the angle

$$\theta_2 = \tan^{-1} (L_2\, \omega/R_2).$$

The instantaneous value of $I_2$ is

$$i_2 = I_2 \sin (\omega t - \pi/2 - \theta_2);$$

the current lagging behind the pressure as given under (*c*) by the angle $\theta_2$.

(*e.*) A counter e.m.f. of self-induction having a maximum value $L_2\omega I_2$ is set up in the secondary 90° behind $I_2$:

$$\text{Back } e_2 = - L_2\, di_2/dt = L_2\omega I_2 \sin (\omega t - \pi/2 - \theta_2).$$

(*f.*) A counter e.m.f. of mutual induction having a maximum value $M\omega I_2$ is impressed on the primary:

$$\text{Back } e_{1s} = - M\, di_2/dt = M\, \omega\, I_2 \sin (\omega t - \pi/2 - \theta_2).$$

In Fig. 6, let $oa = I_1 R_1$ represent the resistance pressure in the primary coil producing the current $I_1$. Draw the induced pressures $M\, \omega\, I_1 = oc$ and $L_1\, \omega\, I_1 = ob$ at right angles to $ao$. At right angles to $I_2$ draw the two pressures $M\omega I_2 = of$ and $L_2\, \omega I_2 = dc$. $ob$, $of$ and

*cd* have been reversed in direction in order to give the impressed e.m.f. on each coil.

The figure gives the pressure relations of the primary and secondary. It is difficult, however, to obtain from this diagram a clear idea of the operation

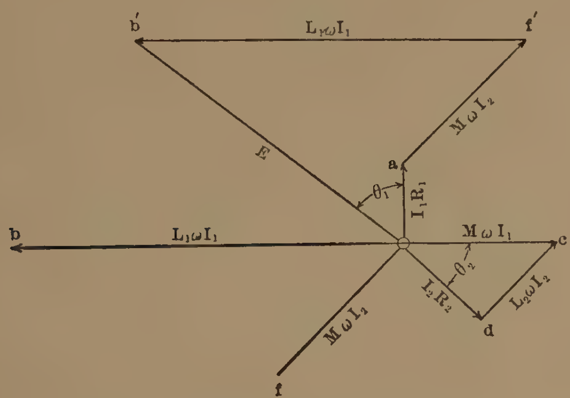



FIG. 6. Simple Transformer Diagram.

of the transformer. The theory of the transformer becomes very much simplified if studied from the point of view of the resultant magnetic flux instead of considering the separate effects of primary and secondary currents.

## CHAPTER III.

### GRAPHICAL REPRESENTATION OF PRESSURE RELATIONS.

**12.** Instead of considering the secondary e.m.f. as being due to the changing current in the primary it will more nearly represent the physical facts to consider it as due to the changing magnetism. This is especially true when the transformer is loaded. When the transformer is loaded, the resultant magnetomotive force is due to the current flowing not only in the primary but also in the secondary. This resultant magnetomotive force produces a resultant magnetic flux, and the secondary e.m.f. should be regarded as due to the change of this resultant magnetic flux. From this point of view the theory of the transformer is simple, and what is learned by studying the transformer can be applied directly to the theory of induction motors, and will assist the student in all alternating current theory. We shall first neglect hysteresis, and assume that all the lines of force which thread one coil also thread the other, that is, that there is no magnetic leakage.

Suppose $i_1$ and $i_2$ be the instantaneous values of the primary and secondary currents. The resultant magnetomotive force acting on the core may be

expressed by the equation m.m.f. $=4\pi\ (S_1\ i_1+S_2\ i_2)$; the resultant magnetic flux therefore, is

$$n=4\pi\ A\mu\ (S_1\ i_1+S_2\ i_2)/l, \qquad (16)$$

and it is the change of this resultant magnetic flux which produces the e.m.f. of the secondary and the back e.m.f. of the primary.

Now consider what takes place in the primary and secondary. In the secondary there is induced an e.m.f. equal to $S_2\ dn/dt$ and this e.m.f., considering the secondary load as non-inductive, is consumed entirely by the resistance of the secondary circuit. Algebraically, since there is no other e.m.f. acting on the secondary circuit, we have, for a non-inductive secondary load,

$$0=(r_2+R_2)\ i_2+S_2\frac{dn}{dt}, \qquad (17)$$

$R_2$ is the resistance of the load and $r_2$ the internal resistance of secondary coil.

If $N$ is the maximum value of the magnetic flux, the maximum value of the e.m.f. induced will be

$$\text{Maximum } E_2=S_2\ N\ \omega=I_2\ (r_2+R_2).$$

If $N=\dfrac{\text{lines of force}}{10^8}$, $S_2\ N\omega$ will be in volts. In the above and in what follows $N=\dfrac{\text{lines of force.}}{10^8}$

Since $E_2$ is induced by the rate of change of $N$, $I_2$ will be 90° behind $N$. Or algebraically, if

$$n = N \sin \omega t$$
$$dn/dt = N \omega \cos \omega t = N \omega \sin (\omega t - \pi/2);$$

which shows that

$$R_2 i_2 = -S_2 \, dn/dt$$

is 90° behind the phase of the magnetism. The relation of the secondary current and the resultant magnetic flux is represented in Fig. 7. (Maximum values are represented.)

In the primary circuit, there is a back e.m.f. acting, due to the change of magnetism in the core of the transformer, equal to $S_1 \, dn/dt$, the maximum value of which is $S_1 N\omega = ab$, and has the same direction as $S_2 N\omega$. The resultant e.m.f. in the primary circuit

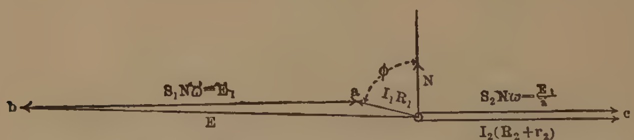



FIG. 7. Pressure Relations on Non-inductive Load $I_2^2 R_2$.

is $R_1 I_1$ and the impressed is $E$. These three pressures must form a closed triangle $oba$. Take $N$ as vertical as in Fig. 7. Now $N$ is produced by the resultant m.m.f.'s of primary and secondary, and, since $I_1$ is 90° behind the resultant M.M.F., $I_1$ must

be in advance of $N$. The angle by which $I_1$ leads $N$ will vary with the load, coinciding with it at no load (assuming no hysteresis) and advancing more and more as the transformer is loaded. For some value of the secondary current, $I_2$, represent the angle by which $I_1$ leads $N$ by Ø, Fig. 7.

Represent $R_1 I_1$ by the line *oa*, and $S_1 N \omega$ by the line *ba*, drawn parallel to *oc*; $E = ob$ must close the triangle *oba*.

$R_1 I_1$ is always small in comparison with $S_1 N\omega$ and therefore $E$ and the secondary pressure $= \frac{E_1}{a}$ are practically in opposition for all values of the load. This fact is of great practical importance in alternating electrical measurements. If $I_2$ equals zero then $I_1$ will coincide with $N$.

**13. Increase of Primary Current with Load.** The resultant magnetomotive force is (neglecting hysteresis) in phase with the resultant magnetism. The primary ampere-turns are in phase with the primary current and the secondary ampere-turns are in phase with the secondary current. Since the


FIG. 8. Increase of Primary Current with Load.

geometric sum of the primary and secondary ampere-turns gives the resultant M.M.F., we have these three quantities related as shown by the triangle *OPR* in Fig. 8.

Now, tests made on good transformers show that when loaded with incandescent lamps the secondary pressure is nearly constant for all loads, the drop in pressure at the secondary terminals being due mainly to the internal resistance. Since *the total secondary pressure* remains practically constant, it follows that the maximum value of the magnetism in the core remains practically constant for all loads, and hence, *oR* remains practically constant for all loads.

Let the vertical line *OR* represent the resultant M.M.F. due to both coils, then the secondary magnetomotive force may be represented by $PR = 4\pi S_2 I_2$, drawn at right angles to *OR*. The primary magnetomotive force will then, for the particular value of the secondary current $I_2$, be represented by *oP*. We have from the triangle

$$4\pi S_1 I_1 = \sqrt{(\text{M.M.F.})^2 + (4\pi S_2 I_2)^2} \qquad (18a)$$

or

$$I_1 = \sqrt{(\text{M.M.F.})^2/(4\pi S_1)^2 + (S_2 I_2)^2/S_1^2}$$

The resultant magnetomotive force *oR* is small in comparison with *PR* for loads above about 25% full load and, therefore, we obtain from (18*a*)

$$I_1 = S_2 I_2/S_1 = \frac{I_2}{a}; \qquad (18b)$$

that is, the primary current is equal to the secondary

current divided by the ratio of transformation. At no load $OR = 4\pi\ S_1 I_0$, $I_0$ being the magnetizing current. At full load the primary current $I_1$ is generally from 50 to 100 times $I_0$. That is, at full load, $PR$ (Fig. 8) is from 50 to 100 times $OR$. The practical use of the transformer is entirely due to the fact that small currents of high potential may be reduced to large currents of low potential and vice versa. It should be noticed that $I_1$ and $I_2$ are practically in opposition.

When there is no magnetic leakage the locus of

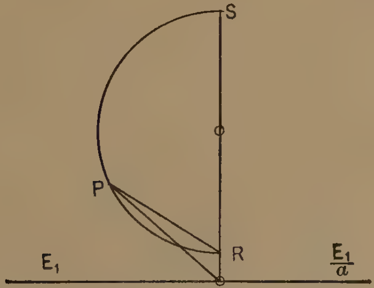



FIG. 9. Locus of Primary Current Vector with Magnetic Leakage Between Primary and Secondary.

the vector $OP$ for non-inductive load is the straight line $RP$, Fig. 8. With magnetic leakage the locus of primary current is the circular arc $RP$, Fig. 9. For open magnetic circuit transformers with large magnetic leakage Fig. 9 is used, the diameter of the circle $RS$ being equal to the primary current which would be obtained by short-circuiting

the secondary at full primary pressure. For closed magnetic circuit transformers $RS$ would be about 1000 times $OR$ and, hence, for this class of transformers the locus of the current may, for working values, be considered as represented by the straight line $RP$, Fig. 8. In the induction motor (the induction motor electrically is essentially a transformer) where there is an air-gap between the primary and secondary a figure similar to Fig. 9 is used.

**14. Effect of Inductive Load on Transformer Diagram.**—If there is an inductive load on the secondary, the secondary current must be drawn to

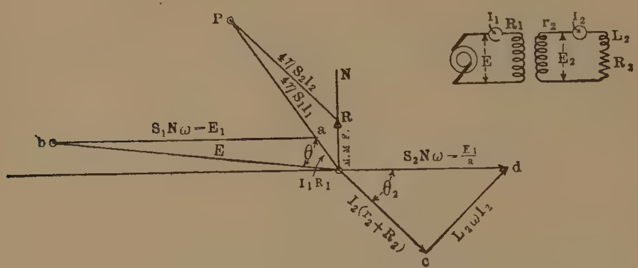


FIG. 10. Pressure Relations with Inductive Load.

show a lag $\theta_2 = \tan^{-1}\dfrac{L_2\,\omega}{R_2 + r_2}$ to get the direction and magnitude of the primary current, Fig. 10. The secondary magnetomotive force must be drawn parallel to $I_2$ from the point $R$, the closing side of the triangle will then be equal to the primary

magnetomotive force. *OR* is small in comparison with *PR* for 25% of full load or over and therefore *oP* and *PR* are practically parallel. That is, the primary current is practically in opposition to the secondary current for a lagging load; also $aI_1 = I_2$, practically.

**15. Effect of Capacity Load on Transformer Diagram.**—If there is capacity in the secondary

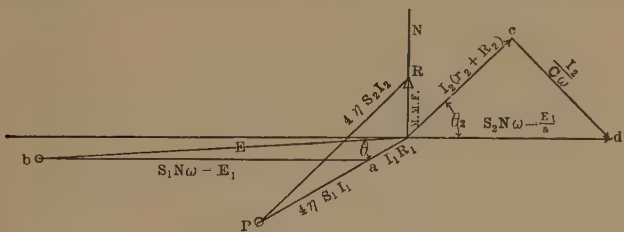



FIG. 11. Pressure Relations with Capacity Load.

circuit the above diagram becomes as shown in Fig. 11. Here the primary current leads the primary pressure. Practically, for 25% full load or over, the power factor of primary and secondary are equal, that is, $\theta = \theta_2$. Hence, $I_2 = aI_1$. For any kind of load, then, $I$ and $I_2$ are practically in opposition.

**16. Effect of Magnetic Leakage on Transformer Diagram.**—The effect of magnetic leakage between primary and secondary windings on the direction of pressure and current in primary and secondary is the same as though the secondary had an inductive

load. The exact effect of magnetic leakage will be explained in another way in the following chapter on "Regulation of Transformers."

**17. The Effects of Hysteresis on Form of Current and Pressure Curves.**—On account of the discrepancy between magnetomotive force and magnetic flux, as shown by the hysteretic cycles, Fig. 12, a sine wave of e.m.f. impressed on a magnetic circuit, will not produce a sine wave of current. (We are here considering only the magnetizing current of

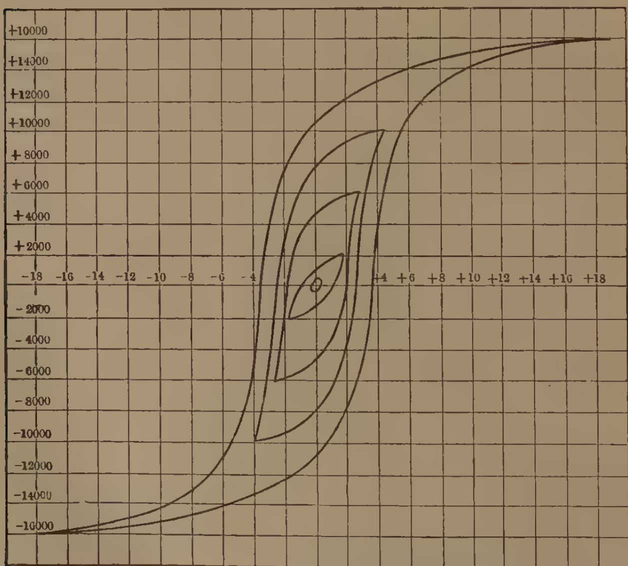
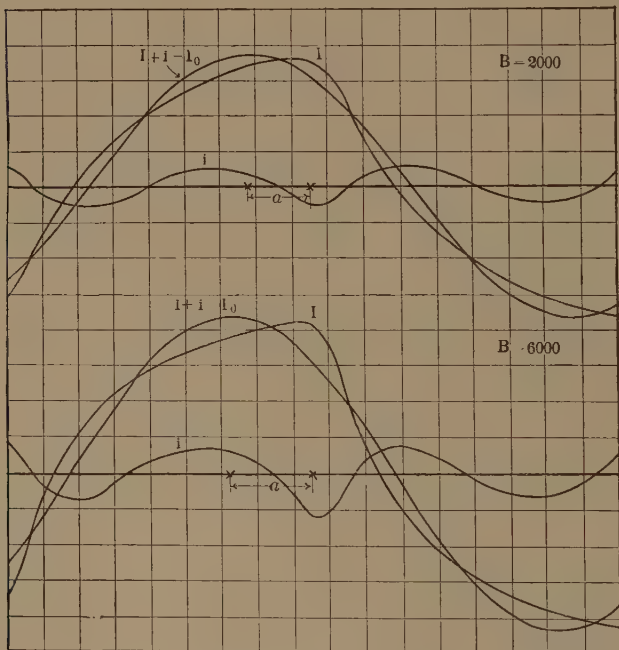



FIG. 12. Magnetic Cycles of Soft Sheet-iron.

a transformer, the secondary being on open circuit.) In the primary circuit we have, as has already been shown, three pressures, the impressed pressure, the pressure $I_1 R_1$ to overcome resistance, and the back pressure $S_1 N\omega$. Now, $I_1 R_1$ is always small in comparison with $S_1 N\omega$, and, therefore, $S_1 (dn/dt)$ and the impressed pressure are nearly of the same magnitude and practically in opposition. Hence, a sine wave of primary pressure implies a sine wave of magnetism, which again implies a sine wave of secondary pressure. But, in order to have a sine wave of magnetism the primary current must differ from a sine wave. To find the form of the current curve to produce a sine wave of magnetic flux, we take from the sine wave of magnetic flux the instantaneous values of $B$ and get from the hysteretic cycle the corresponding values of the magnetomotive force, from which may be calculated the instantaneous values of the primary current. The instantaneous values of current are then plotted as functions of the time. It will be evident from the curve shown in Fig. 12 that the form of the magnetizing current will vary with the maximum value of the magnetic flux. In Figs. 13 and 14 are shown some curves of primary current given by Steinmetz. $I$ is the current curve; $i$ is a curve of current which added to $I$ will give a sine wave of current $I+i$. The curve $I+i$ is the current which would be necessary to produce a sine wave of flux $B$ if hysteresis were absent. As seen from these curves, $i$ is mainly a triple harmonic.

The curve $I$ may be replaced for all practical purposes by the equivalent sine curve $I+i$. The wave of magnetism has its maximum value at the instant




FIGS. 13 and 14. Distortion of Current Wave by Hysteresis.

$I$ has its maximum, and hence the equivalent sine current leads the sine wave of magnetism by the angle $a$; $a$ is called the hysteretic angle of advance.

The exciting current, $I_0 = I + i$, may be resolved

into two components, one in phase with the magnetism and one at right angles to the magnetism; the first is the magnetizing component and consumes no energy, the second is an energy component and must supply the hysteresis loss and is, therefore, in phase with the primary pressure. In Fig. 15, $I_0$ represents the equivalent sine wave of a current $I+i$, $i_m$ is the magnetizing component and $i_h$ is the energy component.

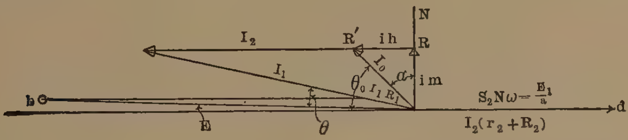



FIG. 15. Effect of Hysteresis on Current and Pressure Relations

It is clear that

$$i_m = I_0 \cos a \text{ and}$$
$$i_h = I_0 \sin a.$$

The product of the primary pressure and the energy component of the primary current, $i_h$, gives the hysteresis energy loss in the transformer. The value of $a$ is usually about 45°, making the power factor of the exciting current about .7 (see Fig. 35).

**18.** Fig. 16 gives the current wave in a transformer at $\frac{1}{10}$ secondary load, showing that at partial load the distortion almost disappears. The magnetic flux is represented by the dotted line.

**19.** If a magnetic circuit has an air-gap the length $i_m$, Fig. 15, will be greatly increased for a given value of $B$, that is, for a given value of $i_h$. Hence, in this

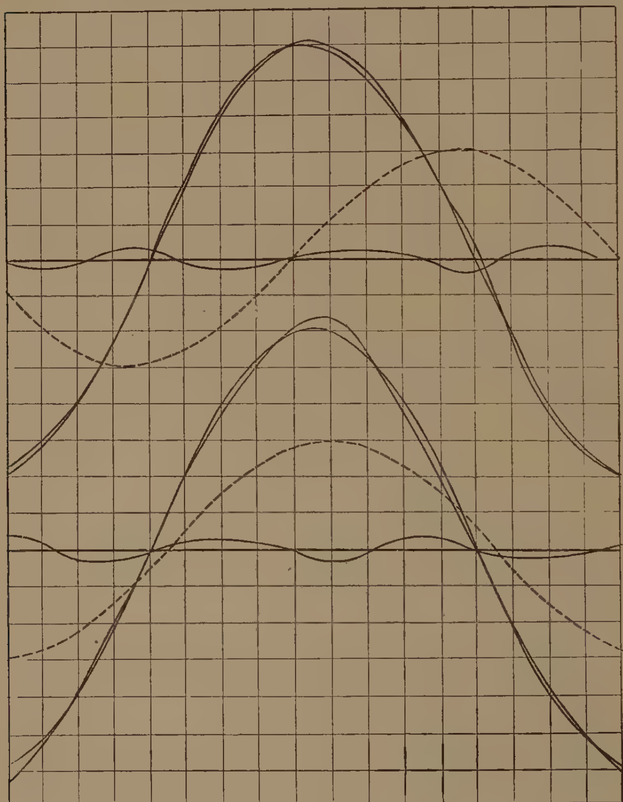


FIG. 16. Current Wave at Partial Secondary Load.
FIG. 17. M. M. F. of Magnetic Circuit with Air Gap.

case, $I_0$ will more or less fall into phase with the magnetic flux and the distortion in the current wave will disappear.

Fig. 17 shows a curve, by Steinmetz, giving the magnetizing current of a magnetic circuit containing an air-gap of 1/400 of the length of the iron.

**20. Relation Between Form of Primary and Secondary Pressure Curves.**—Since the primary pressure applied to the transformer is always practically balanced by the value $S_1\, dn/dt$ and the secondary pressure is also proportional to $dn/dt$, it follows that the primary and secondary pressure waves will always be nearly of the same form.

## CHAPTER IV.

### REGULATION.

**21. First, Neglect Magnetic Leakage.**—The internal resistance of the primary may be considered as connected outside the transformer; similarly, the secondary resistance may be considered as connected between the terminals of the transformer and the load as shown in Fig. 18*a*. $E_1$ is the induced e.m.f., $S_1 N\omega$, in volts. The equation for $E_2$ is

$$E_2 = E_1/a - I_2 r_2.$$

$$\text{But } E_1 = E - I_1 R_1.$$

$$\text{Therefore, } E_2 = E/a - (I_1 R_1)/a - I_2 r_2.$$

In place of $I_1$ put $I_2/a$, and we get

$$E_2 = E/a - I_2 \left(r_2 + \frac{R_1}{a^2}\right). \qquad (19)$$

The second member of this equation gives the pressure at the terminals of the receiver for any value of the current $I_2$. The percentage drop in pressure from no load to full load is

$$\varepsilon = \frac{I_2 \left(r_2 + \frac{R_1}{a^2}\right) 100}{\frac{E}{a}} \qquad (20)$$

When the drop in pressure at the secondary terminals for full load, with constant pressure on the primary is 2%, we say the regulation of the transformer is 2%. Another way of giving the regulation is to give the percentage rise in secondary pressure from full load to no load.

If we multiply equation (19) by $a$ and replace $I_2$ by $aI_1$ we get

$$aE_2 = E - I_1\,(R_1 + r_2\,a^2); \qquad (21)$$

and the percentage regulation of the transformer referred to primary voltage is

$$\varepsilon = \frac{I_1\,(R_1 + r_2\,a^2)\,100}{E}. \qquad (22)$$

This must, of course, give the same value as equation (20).

**22.** From equation (19) we see that, so far as regulation is concerned, the transformer and generator could be replaced by a generator of potential $E/a$, delivering a current $I_2$ over a line having a resistance $r_2 + R_1/a^2$. This is shown in Fig. 18*b*. The value $r_2 + \dfrac{R_1}{a^2}$ is called the *equivalent resistance* of the transformer, considering the same energy as delivered by a generator of potential $E/a$, Fig. 18*b*.

From equation (21) we see that, so far as the calculation of regulation is concerned, the transformer and its load may be replaced by a load to which the

current $I_1$ is delivered, at a potential $E$ over a resistance $R_1 + r_2\ a^2$. This is shown in Fig. 18*c*. The

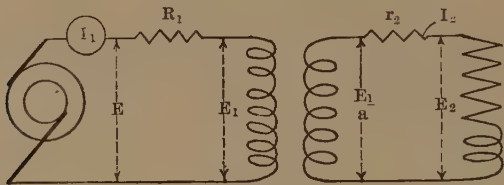



Fig. 18*a*

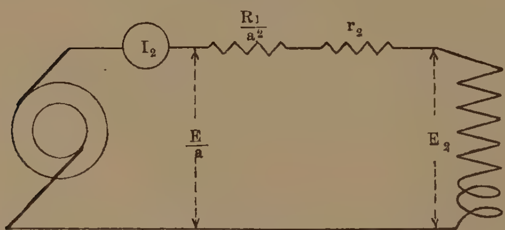



Fig. 18*b*

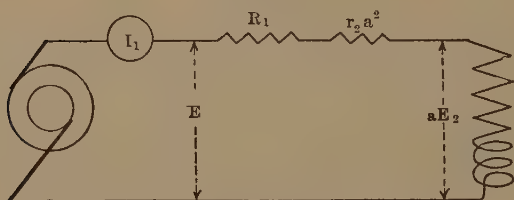



Fig. 18*c*

FIGS. 18*a*, 18*b*, 18*c*. Electrically Equivalent Circuits.

value $R_1 + a^2\ r_2$ is called the *equivalent resistance* of the transformer, considering the same energy transmitted at a potential $E$, Fig. 18*c*. For the purpose

of calculating the regulation the transformer and its load are replaced by an equivalent series system. The method shown in Fig. 18*c* is generally used in practice.

**23.** With no magnetic leakage the only drop in pressure in the transformer is due to resistance. A transformer without magnetic leakage would have less drop on an inductive than on a non-inductive load, due to the resistance drop being out of phase with the delivered pressure. As we shall see, this is not true for practical transformers, all of which have some magnetic leakage. The magnetic leakage of transformers as ordinarily constructed for lighting loads has little effect on the regulation so long as the load is non-inductive. The equations (20) and (22) may therefore be used to find the percentage drop in pressure on a lighting load, for transformers as ordinarily constructed, with little error.

**24. Effect of Magnetic Leakage on Transformer Regulation.**—It has been shown that for a non-inductive load, such as an incandescent lamp load, the pressure at the terminals is given with sufficient accuracy by the equation

$$E_2 = I_2 \; R_2 = E/a - I_2 \; (r_2 + R_1/a^2)$$

In all transformers, however, there is magnetic leakage between the primary and secondary coils which affects the regulation on an inductive or a

capacity load. The following graphical treatment of the effect of magnetic leakage on transformer regulation will make this clear.

In Fig. 19*a*, we have, diagrammatically represented, a transformer having magnetic leakage. Electric pressure is set up in the secondary by a changing magnetic flux which threads the primary and secondary coils. Besides this magnetic flux which threads both circuits, there are lines of force, due to magnetic leakage, which thread only the primary turns, and other lines of force which thread only the secondary turns. Now, consider the effect of the leakage lines of force—that is, those lines of force which thread one coil but do not thread the other—on the pressure relations in the primary and secondary coils.

**25.** The lines of force which thread only the primary turns will, clearly, set up an e.m.f. of self-induction at right angles to the primary current; that is, these lines of force act like an inductive e.m.f. in series between the dynamo and the transformer. The leakage lines in the secondary circuit, similarly, act like an inductive e.m.f. in the secondary circuit between the terminals of the transformer and the load. What amounts to the same thing, we may consider a certain number of the primary turns as taken out of the transformer and put in the line between the dynamo and the transformer. Similarly, a certain number of secondary turns may be considered as connected between the transformer and

the load. Represent the self-inductions corresponding to these turns respectively by $L_1$ and $L_2$, and we have a condition represented diagrammatically in Fig. 19*b*.

A little consideration will make it evident that, in a transformer in which the primary and secondary

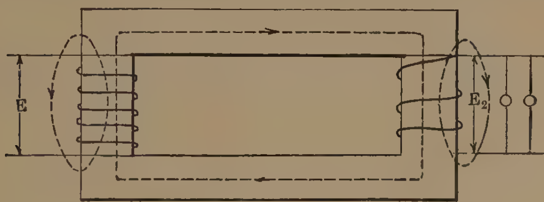



FIG. 19*a*. Transformer with Magnetic Leakage.

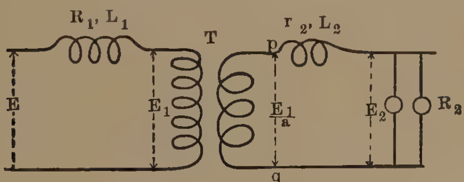



FIG. 19*b*. Electrically Equivalent to 19*a*.

coils are similarly disposed with respect to the magnetic cricuit, the primary turns in series in the primary circuit are equal to the ratio of turns, *a*, on the transformer, times the turns in series in the secondary circuit. For, the leakage lines are nearly equal for the two coils, but the primary leakage lines cut

*a* times the number of turns cut by the secondary leakage lines.

This leaves a transformer, *T*, Fig. 19*b*, with the same ratio of transformation as the ratio of turns, *a*, in Fig. 19*a*, but in which there is no magnetic leakage

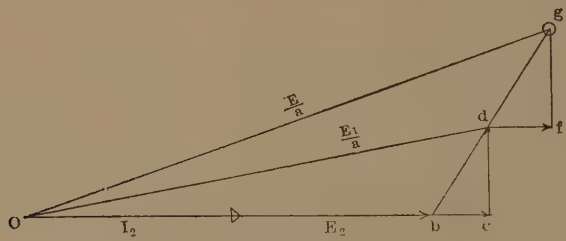



Fig. 20

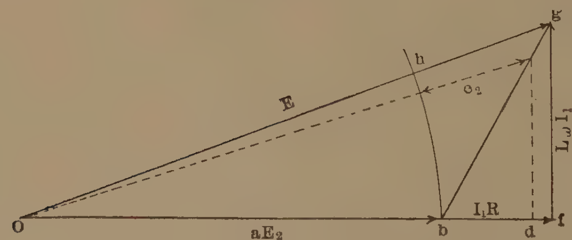



Fig. 21

FIGS. 20 and 21. Pressure Diagrams of Fig. 19*a*.

between the primary and secondary turns. With this arrangement it is an easy matter to determine the regulation of the transformer for any load and for any power factor.

**26.** We will assume for the present that the inductances $L_1\omega$ and $L_2\omega$ are known, and point out later how the regulation is determined in practice without loading the transformer. The transformer is assumed to carry such a load that the primary and secondary currents are practically in opposition. This assumption will be true for any closed circuit transformer carrying 25% load or over. Let us take, first, the case of non-inductive load. Lay off, as in Fig. 20, $ob = E_2$ horizontally—in the same direction as $I_2$, the secondary current. From $b$ lay off the value $bc = I_2 r_2$, the resistance drop in the secondary, and at right angles to $bc$, lay off $L_2 \omega I_2 = cd$. This gives $od = E_1/a$ as the e.m.f. that would have to be applied to $pq$, Fig. 19$b$, to produce the pressure $E_2$ at the terminals of the load. We must now pass to the primary circuit, but instead of multiplying all the sides of the triangle $ocd$ by the ratio of turns, we may reduce all the pressures in the primary circuit by the ratio of transformation, and lay them off on Fig. 20.

Lay off from $d$ the value $df = I_1 R_1/a$ parallel to $bc$, and from $f$ the value $fg = L_1 \omega I_1/a$, at right angles to $df$. This gives $og = E/a$ as the primary pressure, reduced by the ratio of transformation, that must be applied to the primary of the transformer to produce the secondary pressure $E_2$ at the terminals of the load. The value of the resistance and the reactive drops in the transformer have been exaggerated to make the figure clear. In practice, however, by taking a large scale of pressures, very good

results may be obtained by the graphical method. If $ob$ is taken equal to 10 inches good results will be obtained.

**27.** With the assumptions that have been made, $cd$ and $fg$, of Fig. 20, have practically the same value, and since $bc$ and $df$ are small in comparison with these $bdg$ may be taken as a straight line. $bc$ and $df$ are also about equal, usually. We may, therefore, replace the two triangles, $bcd$ and $dfg$, by a single triangle, $bfg$, Fig. 21, having as base a length, $bf$, proportional to the load on transformer. (In transferring to Fig. 21, multiply all the sides of Fig. 20 by the ratio of transformation. This simply means to change the scale of the drawing.) We get

$$bf = (bc + df)\, a = I_2 r_2 a + I_1 R_1 = I_1 (r_2 a^2 + R_1) = I_1 R, \tag{23}$$

in which $R$ is the equivalent resistance of the transformer. If, now, with $aE_2 = ob$ as radius, we draw the arc of a circle $oh$, we cut off between the straight line $bg$ and the arc $bh$ a length $e_2$, which gives the drop in secondary pressure for the load corresponding to the length $bd$. That is, the regulation of the transformer is shown in the figure. Algebraically expressed, we have, from the figure:

$$E^2 = (aE_2 + bf)^2 + (fg)^2, \quad \text{giving}$$

$$aE_2 = \sqrt{E^2 - (fg)^2} - bf. \quad \text{Here} \tag{24}$$

$$fg = I_1 (L_1 w + a^2 L_2 w) = I_1 L\omega \quad \text{and} \tag{25}$$

$$bf = I_1 R.$$

$L\omega$ is the *equivalent inductance* and $R$ the equivalent resistance of the transformer.

In practice the two triangles *bcd* and *dfg*, in Fig. 20, cannot be obtained very easily. The triangle *bfg*, Fig. 21, which answers our present purposes equally as well, is determined by short-circuiting the secondary winding and determining the pressure *bg*, Fig. 21, necessary to send full load current through the transformer.

**28.** The length *fg* is called the inductive pressure in the transformer. If *e* is the voltage necessary to produce full load current with the secondary short-circuited, then *fg* is calculated by the formula

$$fg = L\omega I_1 = \sqrt{e^2 - (I_1 R)^2}, \qquad (26)$$

in which $R = R_1 + r_2 a^2$. The percentage inductive pressure is

$$\frac{(L\omega I_1)\ 100}{E}, \qquad (27)$$

where $E$ is the primary transformer pressure.

**29.** Ordinarily the resistance pressure in transformers varies from .5% to 4%. For small lighting transformers it is usually between 1½% and 3%. The percentage inductive or reactive pressure may vary from 2 to 15. For good lighting transformers the percentage inductive pressure is usually between 3 and 6. In specifying the regulation

of a transformer, both the inductive pressure and resistance pressure should be given in per cent.

**30.** Having the resistance pressure and inductive pressure in the transformer it is an easy matter to find the regulation for any load and any power factor. For any other power factor of the load the line *bf*, representing the resistance drop, must be

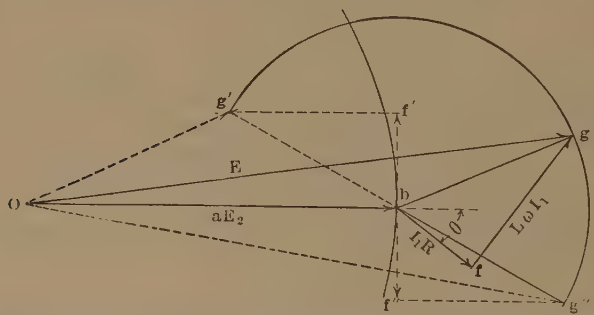



FIG. 22. Regulation of Transformer for Different Power Factors.

drawn from *b* (Fig. 22) in such direction with respect to $E_2$ to correspond to the power factor of the load, and *fg* drawn at right angles to *bf*. In Fig. 22 is shown in heavy lines the position of the triangle for a power factor of .8.

**31.** The locus of the point *g* for the same current but variable power factor is a circle with *bg* as radius and *b* as centre. The limiting positions of the

triangle are $bf'\,g'$ and $bf''\,g''$, corresponding to 90° leading or lagging current respectively. As seen from this figure, for lagging currents the secondary pressure drops, and for leading current the secondary pressure rises, with increasing load. $og'$ multiplied by the ratio of transformation will give the dynamo pressure for a secondary pressure $E_2$ applied to a load having a 90° leading current, and $og''$ the pressure for a 90° lagging current. The rise in pressure for leading current is especially noticeable on long distance transmission lines. The charging current of the line passes through the transformer where a rise in pressure takes place equal to the equivalent reactance of the transformer multiplied by the capacity current.

**32.** To obtain good regulation for all power factors $fg$ must be small. In the best transformers, however, $fg$ is several times $bf$. The general equation for the secondary pressure may be written out by inspection of Fig. 22. The graphical solution, however, is sufficiently accurate for practical purposes and has the advantage that it presents the condition of the circuit to the eye.

By dividing the primary and secondary coils (see Fig. 41), and providing a short magnetic circuit, magnetic leakage may be reduced.

**33.** As an example of the above let us determine the regulation of a transformer, the normal primary pressure being 2200 volts and the full load primary

current 2.5 amperes. The resistance of the secondary measured .02 ohms and the primary 10 ohms. Hence, the equivalent resistance of the transformer referred to primary is

$$10 + (.02)\ (20)^2 = 18 \text{ ohms.}$$

(The ratio of transformation is 20 to 1.) The resistance pressure is (18) 2.5 = 45 volts, or about 2%.

The voltage on primary necessary to produce full load current with the secondary short-circuited was found to be 130 volts. Hence,

$$L\omega I_1 = \sqrt{(130)^2 - (45)^2} = 122 \text{ volts,}$$

or about 5 5%. The maximum drop which this transformer can have on any kind of load for full load current would be 130 volts or about 6%. For non-inductive load the pressure at the terminals of the secondary will be

$$E_2 = \frac{\sqrt{(2200)^2 - (120)^2} - 45}{20},$$

$$= \frac{2200 - 45}{20} \text{ (practically),}$$

$$= \frac{2155}{20} = 107.8 \text{ volts,}$$

or a drop or 2.2 volts from no load to full load. The drop in pressure for a load of any other power factor is most easily determined by constructing a figure similar to Fig. 22, in which $aE_2$ is 2200, $bf = 45$ and $fg = 122$ volts.

# CHAPTER V.

## EFFICIENCY.

**34.** The efficiency of any transformer may be written

$$\text{Eff.} = W_2/(W_2+L) = W_2/W_1 = (W_1 - L)/W_1,$$

in which $W_1$ is the power absorbed by the primary, $W_2$ is the power delivered to the terminals of the secondary, and $L$ is the total loss in the transformer. The power lost in the transformer is made up of the $I^2R$ losses in the primary and secondary, the foucault current losses in the iron and copper, and the power lost by hysteresis in the iron. The $I^2R$ losses are proportional to the square of the load current. The hysteresis and foucault current losses have been found to be practically independent of the load. The losses in the transformer may be represented with sufficient accuracy by the equation

$$L = I_2^2 r_2 + I_1^2 R_1 + C = I_1^2\ (a^2\ r_2 + R_1) + C,$$

$C$ being the hysteresis and foucault current losses. Substituting in the equation for efficiency, we obtain:

$$\text{Eff.} = \frac{(W_1 - I_1^2\ (a^2\ r_2 + R_1) + C)}{W_1}.$$

Now, for non-inductive above about 20% of the

maximum, the energy given to the primary is, practically, $E\,I_1$; that is, the power factor for the transformer may be taken as unity. This gives us:

$$\text{Eff.} = \frac{(EI_1 - I_1^2\,(a^2\,r_2 + R_1) + C)}{E\,I_1}.$$

To determine when this will be a maximum, derive $\frac{d(\text{Eff.})}{dI}$ and equate to zero. The condition of maximum efficiency will be found to be

$$C = I_1\,(a^2 r_2 + R_1).$$

That is, the transformer will be working at its greatest efficiency when the copper and iron losses are equal. When running on a variable load the total watt-hours lost in the iron should be equal to the total watt-hours lost in the copper.

**35.** It does not follow, however, that we must load our transformers so as to produce equality between the copper and iron losses. The pressure-regulation and coolness of the machine impose limitations that must not be ignored. Many large transformers are operated in practice below that given by the rule. If the load were pressed to the point of maximum efficiency, they would either overheat, or would fall off in secondary pressure beyond the proper limits. This only means that their efficiency is already sufficiently high to meet all requirements. The copper loss in lighting transformers is usually

larger than the core loss. The curves given, Fig. 23, which give the efficiency of two standard transformers, show that the efficiency rises very

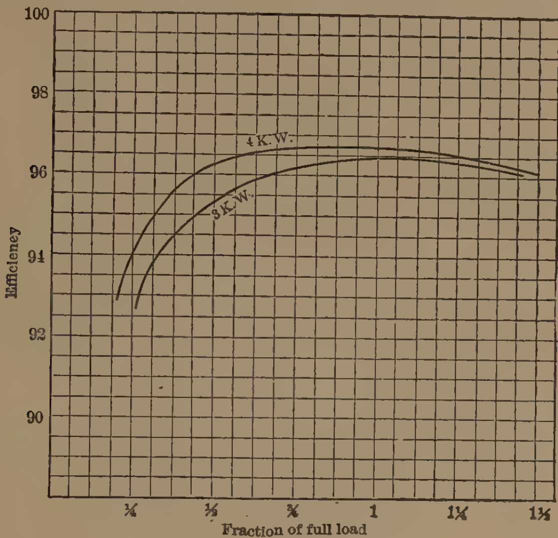



FIG. 23. Transformer Efficiency.

rapidly and attains a very high value at $^1/_2$ and $^3/_4$ load, and that in the neighborhood of full load the efficiency remains practically constant.

**36. All-Day Efficiency of Transformers.**—When a transformer is to be operated on a lighting load, it is important to know not only the total losses in the transformer, but also the proportion between the iron and the copper losses for a given

load; for the transformer giving the best full load efficiency may not be the most economical for the variable load to be carried. The reverse may even be true; the transformer giving the lowest full load efficiency may be the best to use for the given load. For example: Suppose we have two transformers of 1,000 watts capacity, the first having an efficiency of 92% and the second 94%. In the first transformer the core losses are 2% and the copper losses 6%; in the second, the core losses are 4% and the copper losses 2%. The load to be carried is variable, but will be assumed to be equal to full load capacity for 5 hours, the remaining 19 hours the transformers are unloaded. Which of the above will be the most economical?

FIRST TRANSFORMER.

| | | | |
|---|---|---|---|
| Total load on transformer .... | 5,000 | Watt-hrs. | |
| Loss by hysteresis (24 hrs)..... | 480 | " | " |
| Copper loss (5 hrs.)........... | 300 | " | " |
| Energy given to transformer... | 5,780 | " | " |
| All-day efficiency (5000/5780) . | .865 | | |

SECOND TRANSFORMER.

| | | | |
|---|---|---|---|
| Total load on transformer..... | 5,000 | Watt-hrs. | |
| Loss by hysteresis (24 hrs.).... | 960 | " | " |
| Copper loss (5 hrs.)........... | 100 | " | " |
| Energy given to transformer... | 6,060 | " | " |
| All-day efficiency (5000/6060) . | .825 | | |

Here the transformer giving the better full load efficiency gives a smaller all-day efficiency.

**37. Hysteresis.**—Mr. Steinmetz has shown that the energy loss by hysteresis can generally be expressed with sufficient exactness by the formula

$$W = k\,B^{1.6},$$

where $W$ is the loss per cycle in ergs per cu. cm. of iron $B$ is the maximum magnetic induction, and $k$ is the coefficient of hysteresis.

The value $k$ varies with the chemical and physical constitution of the iron. All iron used in transformers should be annealed before assembling in the core. As the hysteresis losses are the most important factors in determining the all-day efficiency, too much stress cannot be laid upon the necessity of careful tests on all transformer iron. Steinmetz has found $k$ to vary between .00124 and .0055, giving an average of .0033. Hysteresis tests on transformer iron are given in Chapter VII.

**38.** The energy lost depends on the maximum value of the magnetic flux, and since, for a given effective pressure, a flat top wave of magnetic flux will have a lower maximum than a sine wave the hysteresis loss will be less because a flat top wave of magnetic flux implies a peaked wave of e.m.f., and conversely. From this point of view a peaked wave of e.m.f. is an advantage. However, there are other disadvantages, the result being that all manufacturers are striving to produce a standard smooth wave. This standard wave for which all are striving is the sine wave.

**39.** The eddy current loss in transformer cores is usually less than one-half the total core loss. This loss varies as the square of the frequency and the square of the magnetic induction. A fuller discussion of the separate losses will be given in the chapter on "Systematic Transformer Design."

**40. Calculation of Efficiency of Transformers.**— If we know the iron losses and the copper losses for full load, it is a simple matter to calculate the efficiency at any load. For example: a 5 K. W. transformer has a core loss of $1\frac{1}{2}\%$, or 75 watts, and a copper loss of 2%, or 100 watts, at full load. (The core losses remain practically constant and the copper losses vary as the square of the load.) At one-half load the copper losses are $(\frac{1}{2}) \times (\frac{1}{2}) \times 100$ equals 25 watts; at any other fraction $1/n$ of full load the copper losses are $(1/n^2)\ 100$. The efficiency, then, at any fraction $1/n$ of full load, is

$$Eff. = \frac{\dfrac{5000}{n}}{\dfrac{5000}{n} + \dfrac{100}{n^2} + 75}, \text{ or}$$

$$= \frac{5000}{5000 + \dfrac{100}{n} + n(75)}.$$

At $\frac{1}{2}$ load $n = 2$, at $\frac{1}{4}$ load $n = 4$, at full load $n = 1$, at $1\frac{1}{4}$ load $n = 4/5$, etc.

At 1/100 load (or when carrying about one 16 c. p. lamp) the efficiency is

$$Eff. = \frac{5000}{5000 + \frac{100}{100} + 100\,(75)} = .40$$

$$Eff. \text{ at } {}^1/_4 \text{ load} = \frac{5000}{5000 + \frac{100}{4} + 4\,(75)} = .939$$

$$Eff. \text{ at full load} = \frac{5000}{5000 + 100 + 75} = .966$$

$$Eff. \text{ at } 1{}^1/_2 \text{ load} = \frac{5000}{5000 + (\frac{3}{2})\,100 + (\frac{2}{3})\;75} = .96$$

The efficiency is a maximum at ${}^3/_4$ load, because at this load the copper losses are equal to the core losses.

**41.** In the early days of alternating current working not much attention was paid to the kind of iron used for transformers, and the iron was not generally annealed. As a result, the core losses were generally very high, perhaps as much as two or three times the loss now allowed in transformers of the same size. A 1 K. W. transformer installed about 1890 shows a core loss of more than 100 watts, or 10%, whereas in a good modern design the loss would not

be greater than 35 watts, or $3^1/_2\%$. There are still some poor transformers put on the market at the present day. A 4 K. W. 60-cycle transformer showed a core loss of 66 watts, another transformer of the same size measured 90 watts. It is highly important, therefore, that central station men test their own transformers. This can be done at little expense. Instructions for making complete tests of transformers are given in Chapter VI.

**42.** The core losses of ordinary 2,000 volt lighting transformers are usually not more than given in the following table:

| K. W. CAPACITY. | PER CENT. CORE LOSS. | K. W. CAPACITY. | PER CENT. CORE LOSS |
|---|---|---|---|
| 1 | 3.5 | 4 | 1.75 |
| 1.5 | 2.65 | 5 | 1.5 |
| 2 | 2.25 | 10 | 1.4 |
| 2.5 | 2 | 15 | 1.3 |
| 3 | 1.85 | 20 | 1.2 |

The percentage open circuit current should be less than twice the percentage core loss. For example, the open circuit current of a 5 K. W. transformer should not be greater than $1.5\% \times 2 = 3\%$ of the full load current of the transformer.

The copper loss will usually not exceed the following values:

| K. W. CAPACITY. | COPPER LOSS IN PER CENT. |
|---|---|
| 1 | 3 |
| 2.5 | 2.5 |
| 5 | 2.25 |
| 10 | 2 |
| 50 | 1.5 |

Some manufacturers work well within the values here given. In larger transformers, the copper loss may be below 1%, making the efficiency at full load over 98%.

**43. The Choice of Size of Transformers.**—When it is the question of the choice of a single transformer to fulfill certain conditions, it is a simple matter to choose the transformer that will give the best all-the-year-round results. Generally, however, the central station must furnish energy over a large area, and the question then of the proper sizes becomes more difficult to answer.

Ten years ago it was the general practice to put up a separate transformer for each house. In the majority of cases the size was about $^1/_2$ K. W. $^1/_2$ K. W. transformers now made have a core loss of about 5%. A transformer of this size made ten years ago would probably have had a core loss as high as 10%.

It is an easy matter to calculate the saving made by substituting large transformers for the single house-transformers, and supplying individual customers

from a net-work of secondary wires. For example, suppose the small transformers having 5% core loss are replaced by the same aggregate capacity of larger size in which the core loss will not exceed 2%. Clearly there is a saving of 3% in the core loss. On each thousand watts replaced, there will be a saving of 30 watts, or

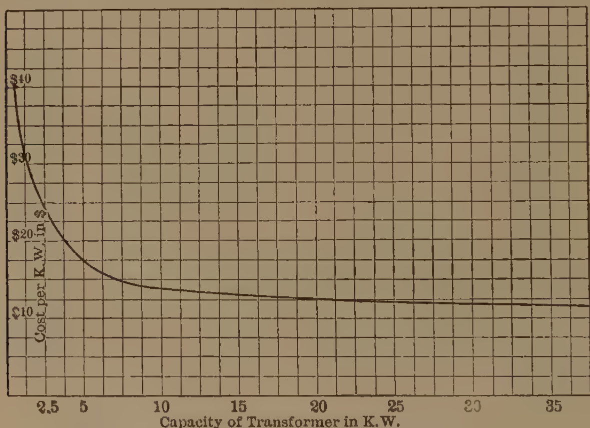



FIG. 24. Curve Showing Cost of Transformers.

$30 \times 8760 = 262{,}800$ watt-hours per year. At two cents per K. W. hour the saving for each K. W. per year would be $5.25. If 1,000 K. W. of the smaller transformers were replaced by the same capacity of larger size, the saving would be $5,250 per year.

There is, of course, the additional saving in line losses and station capacity, due to the reduction of the magnetizing current. At the same time, when

individual customers are supplied from a network of secondary wires, the aggregate capacity is less than if each customer were supplied by an individual transformer.

Further, the cost per K. W. decreases with the size of the transformer, as shown by the curve in Fig. 24. The prices, of course, vary with the market, but the relative prices will be about as given. From the curve we see that the cost per K. W. of about 1 K. W. capacity is about twice that of transformers of about $7^1/_2$ K. W. The cost of an aggregate of 100 K. W. of 1 K. W. capacity will be about $3,000, whereas if $7^1/_2$ K. W. transformers are used the cost will be about $1,500.

**44.** In a paper on alternating current transformers from the station manager's view point before the A. I. E. E. Convention, at the Omaha Exposition, Mr. W. F. White, Vice-President and General Manager of the Omaha Light and Power Company, gave figures showing the saving made in his station by changing from small to large transformers.

A total of 550 transformers, aggregating 1100 K. W., were replaced by about 150 of larger size, aggregating 750 K. W. The saving in coal was about $4,000 per year—the cost of coal being about $2 per ton. During the same time, the number of lamps connected increased from 16,700 to 23,600.

Some incidental advantages of the change are mentioned by Mr. White:

"FIRST. Under the old plan, the 6,941 lights added would have required about 9,200 lights additional transformer capacity, of 40 lights average size, of $1.00 per light average cost, giving an investment saving of $9,200.

"SECOND. The transformer load, with open secondaries, of 80 amperes, was reduced by the Fall of 1897, to about 30 amperes, notwithstanding an increase of about 25% in the number of lights connected. A saving of 50 amperes at 1000 volts represents a capacity to serve, without additional station or line equipment, approximately 1000—16 c. p. equivalents, all burning at one time, or fully 2000—16 c. p. equivalents connected.

"THIRD. Formerly as many as a dozen transformers were frequently burned out in a single thunder storm. Ten per cent. of the transformers connected would be a conservative estimate of the number burned out each year from various causes. Of the new transformers connected during the past three years, only one has failed from any cause, that one was defective and burned out under light load almost immediately upon being put into service. Transformer repair costs are therefore practically eliminated.

"FOURTH. Incandescent lamp renewals have been greatly reduced. Using lamps from the same factory, at the same efficiency, and bought at the same price per lamp, the cost (estimated) of renewals per kilowatt hour consumed by the lamps has been reduced from $.0081 in 1895 to $.0057 in 1897,

a reduction of 29.6%. This saving represents the advantage of regulating the secondary voltage by pressure wires connected to the secondary network, as compared with the former method of regulating the primary voltage by station transformers.

"FIFTH. The principal remaining incidental advantage is the uniformity of voltage, bringing uniformity in quality of service and consequent satisfaction of customers, the value of which cannot be estimated in dollars or percentages."

**45.** At the present time the practice is to divide the area to be supplied by power into districts, feeding each district by a set of transformers, the secondaries being connected in multiple. Each set of transformers is generally fed from the station by a separate line. Certain circuits may be altogether cut out during part of the 24 hours, thus further reducing the losses. The problem of maintaining a uniform voltage over the system is solved by the use of feeder regulators in the line.

**46.** Such a system is much more easily taken care of than the old house-to-house system. The chances of a break-down of insulation varies more than in proportion to the number of transformers, because if we replace 1,000 transformers by ten sets operating on secondary circuits, the insulation between the secondary and primary of each set could be easily tested at stated periods, whereas with the thousand transformers connected it would be

practically impossible to make insulation tests. In other words, we have practically replaced 1,000 poor transformers by 10 good ones.

This matter of making frequent tests of insulation between primary and secondary is an important one, and should be provided for by every station. The station man should know the condition of every transformer on the line, not only to limit the accidents to life and to reduce the fire risk, but also to limit the burn-outs, by removing the defective transformers from the line.

**47. Choice of Transformers for Motor Work.**—Motors are generally arranged so that the transformers are cut out when the motor is not running. Hence, the question of small core loss is not so important as with lighting transformers. For motor work, therefore, such transformers should be used as will give the highest efficiency on the average motor load.

Since motors have a poor power factor, the transformer capacity should exceed the motor capacity. A good rule, which is very generally followed, is to put in one K. W. transformer capacity for each horse-power of motor. That is, for a 10 H. P. motor we should install 10 K. W. transformer capacity.

**48. Aging of Transformer Iron.**—It was first noticed about 1894 that the core loss of transformers increased after they had been in service for a time. The first published reports were probably by Mr.

G. W. Partridge.[1] It was at first thought the effect was due to a kind of magnetic fatigue due to repeated reversals, but it was shown by Mr. Mordey,[2] Professor Ewing, and others to be a temperature effect. The following are the published conclusions of Mr. Mordey:

"FIRST. The effect is not fatigue of the iron caused directly by repeated magnetic reversals—it is not 'progressive magnetic fatigue.'

"SECOND. Neither magnetic nor electric action is necessary to its production.

"THIRD. It is a physical change resulting from long-continued heating at a very moderate temperature.

"FOURTH. It appears to be greater if pressure is applied during heating.

"FIFTH. It is not produced when the iron is not allowed to rise more than a few degrees above the ordinary atmosphere.

"SIXTH. It is similar to the effect produced by hammering, rolling, or by heating to redness and cooling quickly.

"SEVENTH. The iron returns to its original condition on re-annealing.

"EIGHTH. It does not return to its original condition if kept unused and at ordinary atmospheric temperatures, whether the periods of rest are short or long."

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1. The London Electrician, Vol. XXXIV, p. 169.

2. Proceeding Royal Soc., Vol. LVII.

**49.** A series of curves are shown in Fig. 25, taken from a paper read by Mr. S. R. Roget before the Royal Society, May 12, 1898. These curves show—and experiments carried out by other experimenters give the same results—that at about 50° C. (120° F.) the increase in core loss begins to become evident. At a lower temperature no increase takes place.

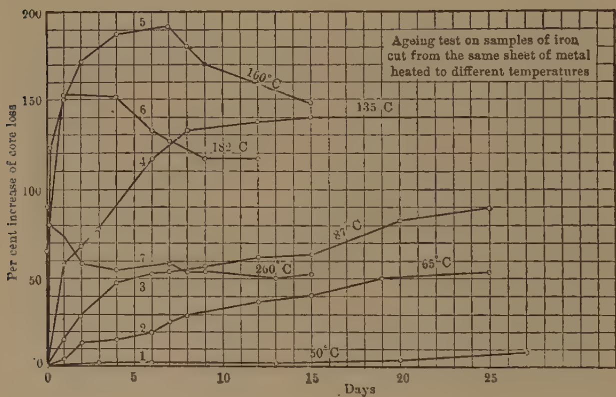


FIG. 25. Aging Test on Samples of Iron, Cut from Sheets of Metal Heated to Different Temperatures.

By properly annealing the iron the increase can be made small below a temperature not exceeding 75° C. (167° F.) Taking the average room temperature as 25° C., or 77° F., this gives an allowable increase in temperature above the surrounding air of about 50° C., or 90° F. As it is the absolute temperature which does the damage, the rise in temperature

must be decreased if the temperature of the air is increased.

In a paper by Arthur H. Ford, presented at the April meeting of the A. I. E. E. in 1900, on "Hysteresis in Sheet Iron and Steel," the author draws

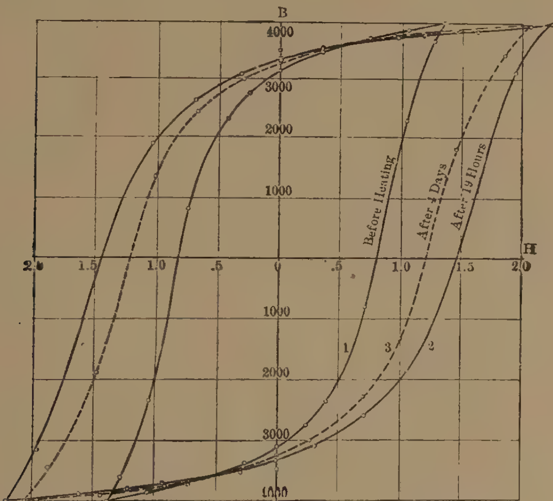


FIG. 26. Showing Change in Hysteresis Loop Due to Aging.

the conclusion, as the result of a large number of experiments, that the hysteresis constant is not so much dependent upon the chemical composition as upon the physical structure of the material due to the method of working.

**50. Methods of Cooling Transformers.**—In small transformers —$^1/_2$ to 20 K. W.—the total iron and

copper losses vary from about 6% to 3%. A 1 K. W. transformer must radiate about 60 watts at full load and a 20 K. W. transformer about 600 watts. On account of danger to insulation or of aging of the iron, it is necessary to limit the temperature rise to about 50° C. above the surrounding air. At a given temperature rise above the surrounding medium the transformer is capable of radiating a given number of watts for each square foot of exposed surface. The square inches of exposed surface should be more than twice the number of watts to be radiated. See Fig. 46.

Small transformers usually have sufficient exposed area to radiate the heat due to the losses. Since the capacity of different sizes increases, however, much more rapidly than the radiating surface, as will be shown in the chapter on transformer design, it becomes necessary in large transformers to provide some artificial means of cooling, notwithstanding the fact that the percentage losses decrease as the size of the transformer increases.

For example, suppose a 1 K. W. transformer has 6% or 60 watts loss at full load, the radiating surface of core and winding being 400 square inches. The watt-loss per square inch of surface is $60/400 = .15$. With this loss per square inch, the temperature rise will not be too great. In a 10 K. W. transformer the losses may be assumed as 3% or 300 watts. The radiating surface, however, will not increase in proportion to the losses. If we assume the radiating surface to be three times that of the 1 K. W., we

have five times the losses with three times the radiating surface, or the watt-loss per square inch will be $300/1200 = .25$. This will give too great a rise in temperature unless the transformer case is filled with oil.

If we increase the size of the transformer to, say, a 50 K. W., we find that in order to get an economical design, special precautions must be taken to insure low temperature in operation. Where a number of transformers are located near together a forced current of air may be used for cooling.

**51.** At the present time, transformers are nearly always enclosed in cast iron cases, filled with a special oil to cover core and windings. The natural circulation of the oil tends to equalize the temperature differences of the various parts, and carries the heat to the case from which it is dissipated. The volume of oil also absorbs heat and, hence, the temperature rise of an oil-filled transformer is less rapid than when only air cooled. Filling the case with oil also improves the insulation, disruptive discharges taking place through it much less readily than through air. If a disruptive discharge does take place, the oil immediately seals up the fault, unless the discharge continues so long that charring occurs. The only objection to oil is that it increases the liability of fire, but this has been proved by practical experience to be an objection of no great importance. Large oil-filled transformers should be isolated as much as possible.

Very large transformers generally have cases filled with oil and, in addition, are cooled by a forced current of water which is carried around inside the transformer case in coils. The water is forced through at such a rate as to keep the temperature down to the desired value. One gallon of water per minute raised through 4° C. (7.2° F.) will carry away approximately 1 K. W. dissipated in heat.

Before filling a case with oil the core and windings should be heated (this may be done by applying a pressure to one of the windings) to drive out the moisture. In large transformers the extra precaution is usually taken to pump the air out of the case and the oil then introduced with partial vacuum in the case.

In some cases where water is expensive transformers are cooled by a forced circulation of air.

# CHAPTER VI.

## TESTING.

**52.** Manufacturers, a few years ago, did not encourage independent testing of transformers by station men, partly because the methods required more skill than the average station man possessed. At the present time, however, the methods of making transformer tests have been reduced to such simple operations that all the leading manufacturers invite tests by the station man. The General Electric Company has issued an excellent little pamphlet on transformer testing for central station men, in which instructions are given for making tests. Some of the illustrations in this chapter are taken from the pamphlet.

The following tests will suffice to give the complete characteristics of a transformer:

(*a*.) Insulation;
(*b*.) Regulation;
(*c*.) Core loss and exciting current;
(*d*.) Copper loss;
(*e*.) Rise of temperature of transformer on full load;
(*f*.) Ratio, and polarity test.

**53.** (*a*.) Above all things a transformer must be

safe, and a test of insulation should first be made. For the insulation test an alternating e.m.f. is applied between the primary and core, between the primary and secondary, and between the secondary and core. For 2,000 volt lighting transformers the

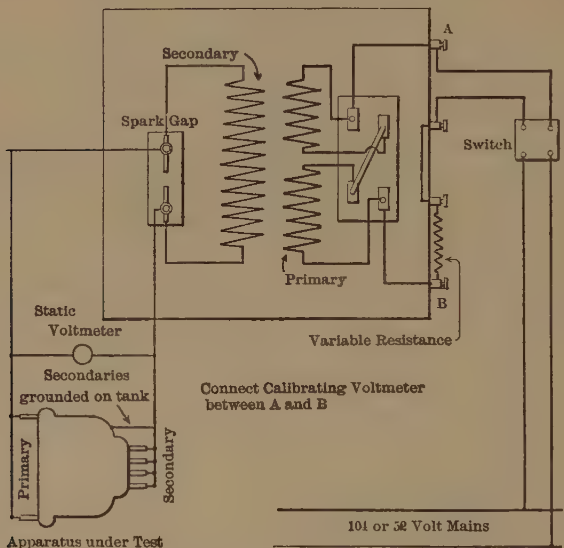


FIG. 27. Testing Insulation of Transformers.

voltage between primary and core, and between primary and secondary is 10,000 (this is a requirement of the insurance companies), between the secondary and core a voltage of about 2,500 should be applied. Measurements of insulation resistance

are not reliable. Fig. 27 gives the connection for the high voltage test between primary, and core and secondary. All primary wires should be connected together as well as all the secondary wires. In testing between the primary and core and

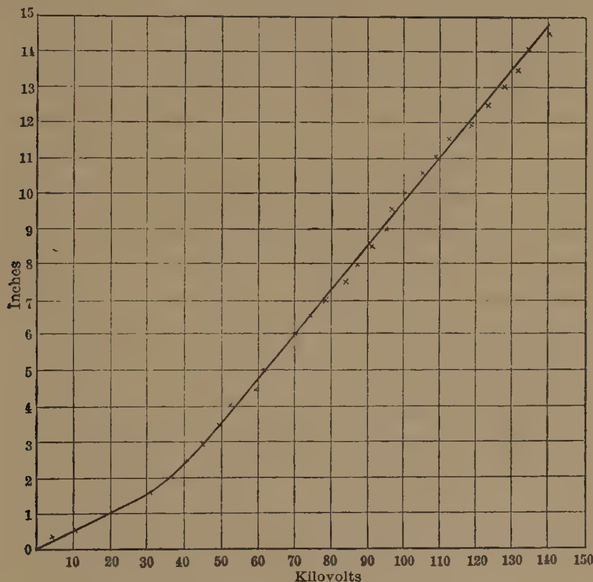


FIG 28. Calibration of Spark Gap.

between the primary and secondary, the secondary must be connected to the core, otherwise the secondary will be subjected to strains not received in practical operation. In Fig. 28 is given the calibration curve of the spark gap. Care should be taken

to have clean needle points and to use new needles after every discharge. The static volt-meter is not needed if the spark gap is set for the correct voltage. Multiplying the reading of the volt-meter connected across *A B*, Fig. 27, by the ratio of transformation will give the primary voltage within a few per cent., provided there is no break-down. The high voltage testing transformer is usually made with primary and secondary on opposite sides of the core, so that in case of faulty insulation in the unit under test the current drawn through the transformer will cause its voltage to drop so much that it will not be injured.

The high voltage should not be suddenly applied or removed. Increase and decrease the voltage gradually until a spark is obtained across the spark gap or the static volt-meter reads the desired value. The high voltage should not be long continued, as the continued strain may permanently weaken the insulation. There will be a certain capacity current necessary to charge the transformer on test as a condenser, but the difference between this and a leakage current can be determined with a little experience.

**54.** (*b.*) For small lighting transformers the regulation may be determined by loading with incandescent lamps and measuring the percentage drop from no load to full load. For large transformers the regulation is calculated as given under "Regulation of Transformers."

The resistance of primary and secondary may be measured by any of the methods used for determining a resistance. Having the primary and secondary resistance, the percentage $IR$ drop may be calculated from equation (20) or (22), Article 21. To correct for temperature, add 1% to the resistance for each 5° F. above which the resistance is measured.

To get the percentage reactive pressure we short-circuit the secondary as shown in Fig. 29, and

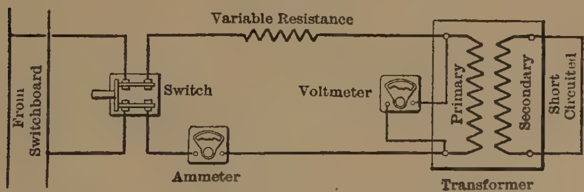


FIG. 29. Determination of Impedance (Note that the Frequency is Correct).

determine the pressure necessary to force full load current through the transformer. For medium sized 2,000-volt lighting transformers the reactive pressure is usually between 3% and 6%. That is, with the secondary on short circuit, 60 to 120 volts applied to the primary will give full load current. The variable resistance shown in Fig. 29 is not necessary if about 100 volts are available. By measuring the current which this voltage produces, we may easily interpolate the voltage necessary to

produce full load current. We now have the percentage pressure consumed by the resistance (this may be determined from the current and the equivalent resistance as explained), and by the impedance. The reactive pressure may be determined graphically as follows: Lay off the percentage resistance pressure, say 2%, horizontally, Fig. 30. From *f* draw a line, *fg*, of indefinite length, at right angles


FIG. 30. Determination of Reactive Pressure.

to *bf*. Then with *b* as centre, and a radius equal to the percentage pressure consumed by impedance, describe a circular arc to cut *fg* in *g*. The reactive pressure may be scaled off, or may be calculated by solving the right angle triangle. Having the percentage resistance and reactive pressure the regulation of the transformer on a lighting load may be determined by drawing a figure like 21, or for

inductive or capacity load by drawing a figure like 22.

In determining the regulation of all fairly good transformers on any kind of a load the magnetizing current may be neglected. This is true because the magnetizing current is usually only about 2% of the full load current, and the addition of this to the load current only affects the ratio of transformation by a small fraction of a per cent. Besides, the magnetizing current is a constant and does not affect the regulation.

The regulation of transformers that are to be used as step-up and step-down transformers on long distance transmission lines or on motor loads should always be determined by getting the reactive as well as the ohmic drop. Usually the reactive pressure is from two to ten times as large as the pressure to overcome the resistance, and since the drop in pressure on a lagging load, or the rise in pressure on a capacity load, is practically proportional to the reactive pressure it is important to know what percentage of the line pressure this will be. A transformer may give perfect satisfaction so far as regulation is concerned on a lighting load, and be an entire failure when supplying light and power at the same time. Large transformers (about 1,000 K. W.) have been built, giving a resistance pressure as low as .5% and a reactive pressure as low as 3%. The reactive pressure may, however, be 10 to 30%. Of course, such transformers would not be permissible where the lighting and motor load is carried by the

same transformers, as is the case with step-up and step-down transformers on transmission lines.

**55.** (*c.*) The core loss is made up of the hysteresis loss and the loss due to eddy currents. The loss due to these causes has been found to be practically independent of the load. The formula for the hysteresis loss has already been given, Art. 37.

Since eddy currents are true secondary currents, the induced currents will be proportional to the maximum magnetic density and to the frequency. The loss of power, therefore, will be proportional to the square of the magnetic density and the frequency. That is

$W = C\ (fB)^2$, in which
$W$ = energy loss at $f$ cycles per second,
$B$ = maximum magnetic flux,
$C$ = a constant.

In well designed transformers, the foucault current loss is small. Usually the eddy current loss is about 30% of the total. The losses may be separated by the method given in the following chapter.

The total core loss is most easily determined by measuring the power input at full load pressure with no load on the transformer. Connections are usually made as in Fig. 31. Normal secondary voltage should be applied to the secondary with the primary

open-circuited. Knowing that the frequency is correct take simultaneous readings of volts, current and watts. The core loss should not exceed the values given in Article 42. The core loss for a 5 K. W. transformer should be about $1^1/_2\%$, or 75 watts. An ordinary 150-watt watt-meter will be of about the correct capacity where the transformers are not too large. Where there are two 100-volt coils on the secondary we may apply 200 volts, but put the potential coil of the watt-meter across one of the secondary coils, and then multiply the watt-

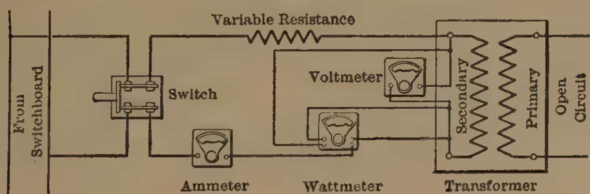


FIG. 31. Measurement of Core Loss.

meter reading by two. In this way the core losses of transformers as large as 20 K. W. may be measured with a 150-watt watt-meter.

The exciting current in percentage of full load current should be less than twice the core loss in percentage. That is, if the core loss is $1^1/_2\%$ the exciting current should be less than 3%. If the secondary full load current is 50 amperes, the exciting current should be less than $1^1/_2$ amperes. In this way the capacity of the ammeter required may be estimated.

The power factor of the transformer on open circuit may be obtained from the ratio:

$$\frac{\text{Watts core loss}}{\text{Volt-amperes Input}}.$$

The power factor at no load is generally about .7.

As previously pointed out, the core loss will be affected by the shape of the e.m.f. wave.

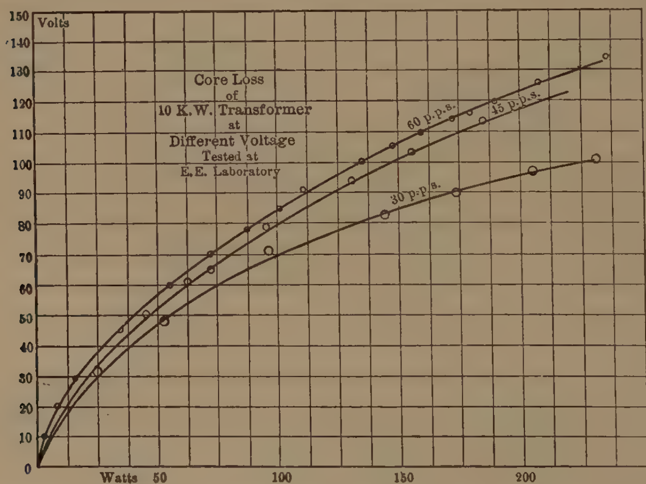


FIG. 32. Shows Curves for the Core Loss of a 10 K. W. Transformer.

Fig. 32 shows curves for the core loss of a 10 K. W. core type transformer at different voltages, for three different frequencies. The transformer was designed to operate at 60 p. p. s., but could be operated at a higher frequency, or as low as 45 cycles. At

30 cycles there is a great increase in core loss. At 60 cycles and 110 volts this transformer operated at an induction of about 9,000 lines per square centimeter. Lowering the frequency to 30 increases the density so much that the magnetic distribution becomes unequal, and the increase in density in certain parts of the magnetic circuit causes the eddy current loss to become large. This accounts for the divergence of the curve for 30 cycles from those of the other two.

**56.** (*d.*) The copper loss may be determined by calculation of the resistance of primary and secondary and the current carrying capacity of the transformer. If $I_2$ is the secondary current, and $R_1$ and $r_2$ the primary and secondary resistances respectively, then the copper loss in watts will be

$$W = I_2^2\left(r_2 + \frac{R_1}{a^2}\right),$$

$a$ being the ratio of transformation. Having the copper and iron losses the efficiency curve of the transformer may be calculated. For small transformers the efficiency may be obtained by taking the ratio of power output to input on load. The efficiency curves of large transformers are nearly always calculated.

**57.** (*e.*) The temperature rise of small transformers may be obtained by loading. With small

transformers, constant temperature will be reached in about eight hours; large transformers will take much longer. The usual requirement for lighting transformers is that after operating for eight hours the temperature rise shall not be over 50° C. or 90° F. The temperature rise should be determined from the oil by thermometer, and by measuring the increase in resistance.

To load large transformers until constant tem-

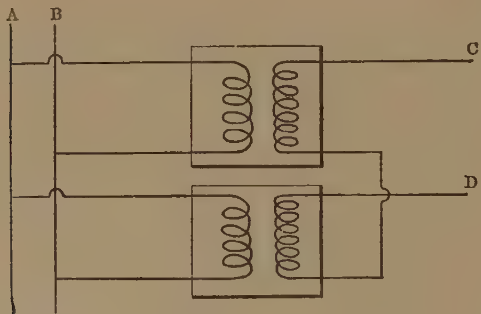


FIG. 33. Artificial Loading of Transformers.

perature is reached would mean a great loss of power. To avoid this, the transformer is usually supplied with a pressure higher than normal—a pressure that will give a core loss practically equal to the combined copper and iron losses when working under full load. This does not give the heating as in practice. Sometimes when the transformer has several coils, it is possible to supply the core loss over one winding with the other windings

connected in opposition and a direct current forced through them. This more nearly approaches practical conditions.

If we have two large transformers to test we may get full load conditions in each and only supply the losses in the two transformers by connecting as in Fig. 33. The secondaries are connected in parallel across the busbars *A B*, and the primaries are connected in opposition. We apply, now, to *A B* a pressure equal to normal secondary pressure of the transformers. To *C D* we apply a voltage equal to double the voltage necessary to produce full load current in one transformer on short circuit. This will give us full load current in both windings in both transformers and normal core loss in each. The circuit *C D* supplies the copper losses and the circuit *A B* the core loss

**58.** (*f*.) Ratio of transformation is checked by a transformer with known ratio.

The polarity may be checked by connecting the secondary in series with that of another transformer and determining by volt-meter if the connection is series or opposition. A small fuse wire may be used in place of the volt-meter; if the connection is series the fuse will be ruptured.

**59.** Mr. R. F. Schuchardt, in an excellent article in the *Electrical World and Engineer*, May 17, 1902, gives the results shown in the table below, as the average values obtained from commercial

transformers at the present day. The voltage to produce full load current with the transformer given in column "Impedance Drop, Per Cent.," varies between 3.4 and 5%. The "Leakage Current" is the exciting current at primary voltage. The "Regulation" is the regulation on non-inductive load. I judge from the data that the tests were on core type transformers.

| Make | Number | Type | Capacity in Watts. | Ratio of Transf. | Date of Test | Pri. Res. Coils in Series | Sec. Res. Coils in Par. |
|---|---|---|---|---|---|---|---|
| x x x | x x x x x 1 | x | 1,000 | $\frac{2080}{1040}=\frac{115}{230}$ | 4-15-02 | 79.0 | .1352 |
| | 2 | | 1,500 | " " | " | 44.1 | .0906 |
| | 3 | | 2,000 | " " | " | 35.0 | .0591 |
| | 4 | | 2,500 | " " | " | 24.0 | .0490 |
| | 5 | | 3,000 | " " | " | 18.0 | .0440 |
| | 6 | | 4,000 | " " | " | 14.4 | .0300 |
| | 7 | | 5,000 | " " | " | 10.6 | .0200 |
| | 8 | | 7,500 | " " | " | 6.50 | .0156 |
| | 9 | | 10,000 | " " | " | 4.61 | .0114 |
| | 10 | | 15,000 | " " | " | 3.04 | .0078 |
| | 11 | | 20,000 | " " | " | 2.01 | .0059 |
| | 12 | | 25,000 | " " | " | 1.72 | .0048 |
| x x x | x x x 13 | x | 30,000 | " " | " | 1.38 | .0035 |

DATA OF TRANSFORMER TESTS.

| | Impedance Drop Percent | Copper Loss Watts | Core Loss Watts | Leakage Current | P. F. at No Load Percent | Regulation Percent | Full Load Efficiency Per cent. | Room Temperature Deg. C. | Remarks |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 4.9 | 31.0 | 30 | .0222 | 65.0 | 3.04 | 94.25 | 23 | |
| 2 | 3.6 | 41.0 | 35 | .0259 | 65.0 | 2.75 | 95.18 | 23 | |
| 3 | 3.7 | 53.0 | 45 | .0332 | 65.0 | 2.60 | 95.33 | 23 | |
| 4 | 4.0 | 60 | 50 | .0370 | 65.0 | 2.38 | 95.79 | 23 | |
| 5 | 3.9 | 70 | 56 | .0414 | 65.0 | 2.32 | 95.97 | 23 | |
| 6 | 3.8 | 92 | 65 | .0480 | 65.0 | 2.28 | 96.24 | 23 | |
| 7 | 4.1 | 102 | 74 | .0547 | 65.0 | 2.06 | 96.60 | 23 | |
| 8 | 3.7 | 150 | 100 | .0740 | 65.0 | 2.00 | 96.78 | 23 | |
| 9 | 3.4 | 192 | 130 | .0960 | 65.0 | 1.92 | 96.88 | 23 | |
| 10 | 3.6 | 296 | 165 | .1220 | 65.0 | 1.98 | 97.00 | 23 | |
| 11 | 3.5 | 365 | 190 | .1405 | 65.0 | 1.95 | 97.02 | 23 | |
| 12 | 3.4 | 480 | 210 | .1554 | 65.0 | 1.93 | 97.32 | 23 | |
| 13 | 3.4 | 525 | 250 | .1850 | 65.0 | 1.77 | 97.48 | 23 | |

DATA OF TRANSFORMER TESTS.

## CHAPTER VII.

### SYSTEMATIC DESIGN.

**60.** The method here given for the design of transformers is based on the line of reasoning first pointed out by Mr. Gisbert Kapp. The attempt has been made to put the matter in such form that a designer may arrive quickly and certainly at the best transformer to meet given conditions, when certain fixed conditions regarding transformer punchings available, etc., are given. It is believed this method has some advantages over that usually given. Methods similar to this are no doubt used by designers of transformers for the manufacturers, but for commercial reasons these are not published.

In designing a transformer, we should strive for:

(*a.*) High insulation;

(*b.*) Good regulation;

(*c.*) High efficiency (small core loss for high all-day efficiency);

(*d.*) Small open-circuit current;

(*e.*) High power-factor on open circuit;

(*f.*) Small rise in temperature;

(*g.*) Low first cost.

(These are not given in the order of importance because the order depends on the conditions to be fulfilled.)

In addition to the above, usually the following fixed conditions are given: the capacity, the primary or secondary voltage, the ratio of transformation, and the frequency. There are then to be determined (1) the number of turns and cross-section of copper, (2) the amount and quality of the iron, (3) the arrangement of copper and iron parts. The qualities of the transformer as given under $a$, $b$, ...... $g$, will, therefore, be determined by the last three variable elements. Some of the conditions, $a$, $b$, ...... $g$, are opposed to others and a compromise must be made to give the best all-round transformer. For example, if we strive for good regulation, we will increase the core loss or have poor insulation. Before beginning the design we should have a test of the iron which is to be used.

**61. Testing Transformer Iron.**—Figs. 34 and 35 give some curves of the properties of annealed sheet transformer iron of two prominent manufacturers. The thickness of the iron was about 14 mils, the gauge ordinarily used for transformer work. Curves between $B$ and permeability, $B$ and watts lost per pound of iron at 60 p. p. s. are given. These samples of iron are probably as good as can be obtained at the present time. For any other frequency multiply the losses given in the curves by $f/60$.

In making the tests a transformer core of ten

pounds of iron was built up, and on this two windings were placed, one of one hundred turns for the current coil, and one of three hundred turns for the pressure

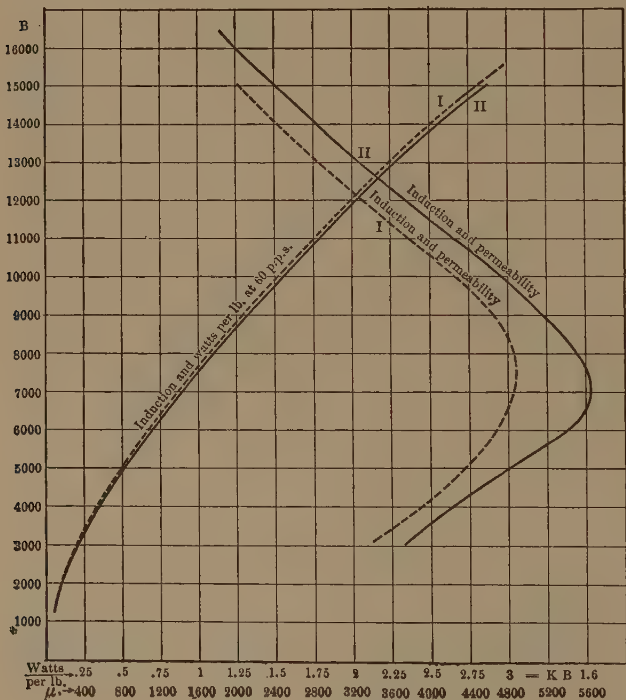


FIG. 34. Tests of Transformer Iron.

coil. The size of the stampings was 2 inches x 4 inches, and these were built up into a rectangular core having outside dimensions of 6 inches x 6

inches, the cross-section of the core assembled being 2 inches x about $1^1/_2$ inches. Simultaneous values

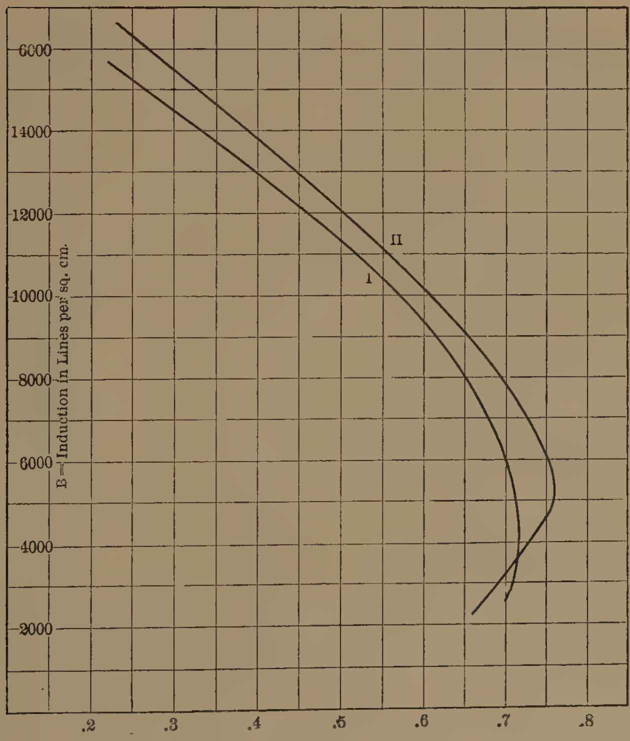


FIG. 35. Power Factor (=cos θ) of Magnetic Circuit. Tests of Transformer Iron made in E. E. Laboratory.

of current to magnetize the iron, the watts lost in the iron and voltage supplied were measured, and

from these readings the curves were derived. In order to get accurate comparative results, the windings were fixed relative to a frame, which received the iron, so as to insure that each sample was being tested under the same conditions. Care was also taken to have the frequency and wave form the same in each case. The wave form was practically a sine wave.

This is believed to be a very good method to obtain accurate comparative results of the magnetic properties of iron. The primary current winding should be about No. 16 and the potential winding about a No. 20 wire, in order to introduce no error due to resistance.

The value of the permeability was calculated from the ratio $B/H_m$, in which $H_m$ is the magnetizing component of the energizing current. That is, $H_m$ was calculated from the formula

$$H_m = \frac{4\pi S i_m}{10\, l} = \frac{4\pi S I_0 \operatorname{Sin} \theta}{10\, l}.$$

(See Fig. 15). The values of Sin $\theta$ were obtained from the power factor as determined by the wattmeter, ammeter, and volt-meter. $B$ was determined from the equation

$$E = \sqrt{2}\, \pi S A B f / 10^8,$$

$E$ being the e.m.f. impressed on the $S$ turns. For the particular case $B = 260 E$; $H_m = 4.38\, I_0 \operatorname{Sin} \theta$.

**62. Separation of Core Losses.**—Several methods have been used for separating the core losses. The method here given will be found convenient where the alternator may be run at different frequencies. Fig. 36 shows the curves for the core loss of a 4 K. W. transformer at different voltages and at three different frequencies. This transformer was designed

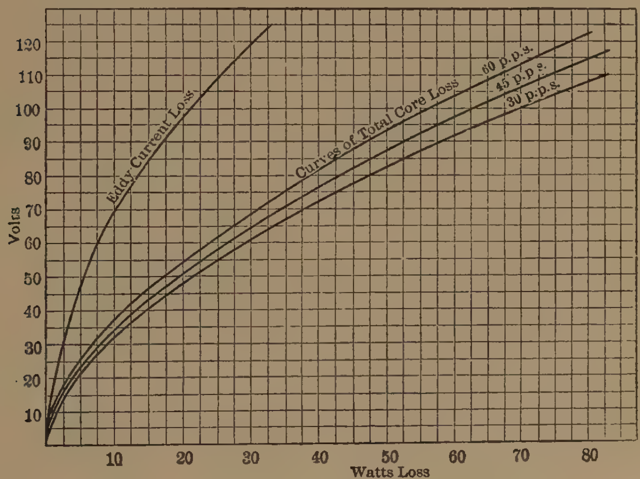


FIG. 36. Core Losses of 4 K. W. Transformer.

for 60 and 120 cycles, but the density at 60 p. p. s. being about 6,000 lines per square centimeter, it may be operated at 30 cycles without saturation, and hence the curve for 30 cycles does not bend to the right as does that shown in Fig. 32.

The striking thing about these curves is the small

increase in the loss for the same voltage when the frequency is decreased. For *the same voltage applied* the eddy current loss remains the same, if the distribution of magnetism remains the same. The increase for the same voltage then represents the increase of the hysteresis loss. If Steinmetz's law holds, the hysteresis loss at 30 cycles should be about 1.5 times that at 60 cycles, as may be shown as follows:

$$W_{60} = F + H_{60} = F + k\ 60B^{1.6} = F + k\ (60B)B^{.6}.$$

$F$ is eddy current loss and $H$ the hysteresis loss.

If now we reduce the frequency one-half, we double the induction. Hence we have,

$$W_{30} = F + k\ (30B2)\ (2B)^{.6} = F + k\ (60B)\ B^{.6}2^{.6}$$
$$= F + H_{60}2^{.6} = F + 1.5\ H_{60}\ \text{(practically)}$$
$$\therefore W_{30} - W_{60} = .5H_{60},\ \text{or}\ 2(W_{30} - W_{60}) = H_{60}.$$

That is, if the law $B^{1.6}$ holds, then multiplying the difference between the core losses at 60 and at 30 cycles, at a given voltage, by two, should give the hysteresis loss at 60 cycles. The losses on the transformer measured, at 60 cycles and 100 volts, was $56^1/_2$ watts, and at 30 cycles 70 watts. The difference is $13^1/_2$ watts. Multiplying this by two gives us 27 watts for the hysteresis loss at 60 cycles, leaving $29^1/_2$ watts, or more than one-half to be accounted for by the eddy current loss. This does not seem possible from an examination of the curves, and therefore we suspect that the exponent 1.6 is too large for this iron.

If we measure the core losses at three different frequencies, the exponent of $B$ may be determined.

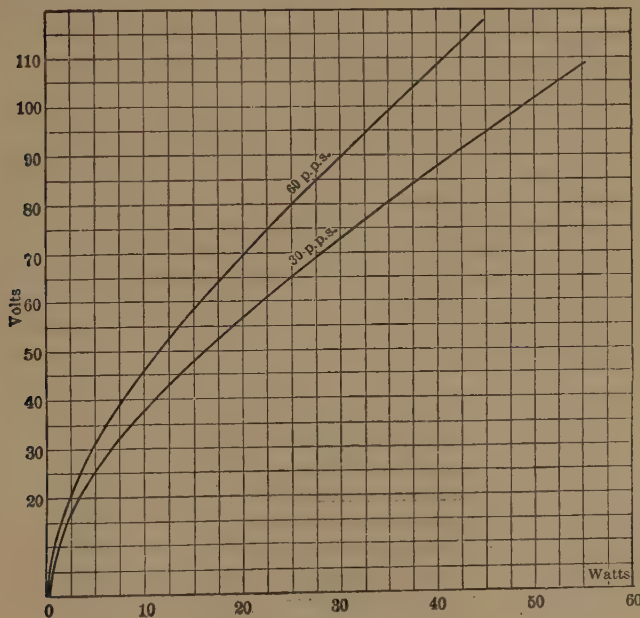


FIG. 37. Hysteresis Loss.

Writing the exponent $1 + a$, we obtain by proceeding as before, the following equations:

$$W_{30} - W_{60} = H_{60}\,[(60/30)^a - 1] = H_{60}\,(2^a - 1).$$

$$W_{45} - W_{60} = H_{60}\,[(60/45)^a - 1] = H_{60}\,(1.33^a - 1).$$

Or $$\frac{W_{30} - W_{60}}{W_{45} - W_{60}} = \frac{2^a - 1}{1.33^a - 1}.$$

Taking the average of a number of readings at different voltages, the left hand member is found to be about 2.1, which makes the coefficient a little less than 1.5 instead of 1.6 as given by Steinmetz's formula. If we assume 1.5 to be approximately correct, we get the equation:

$$2.5\ (W_{30} - W_{60}) = H_{60}.$$

In Fig. 36, the curve of eddy current loss found in this way is shown. Plotting the hysteresis loss on a separate sheet we obtain Fig. 37.

## TYPES OF TRANSFORMERS.

**63.** There are two types of transformers coming into general use, the shell type and the core type.

FIG. 38. Core Type Transformer.

Fig. 38 shows a typical example of the core type, and Fig. 39 one of the shell type. In the two types the copper and iron change places.

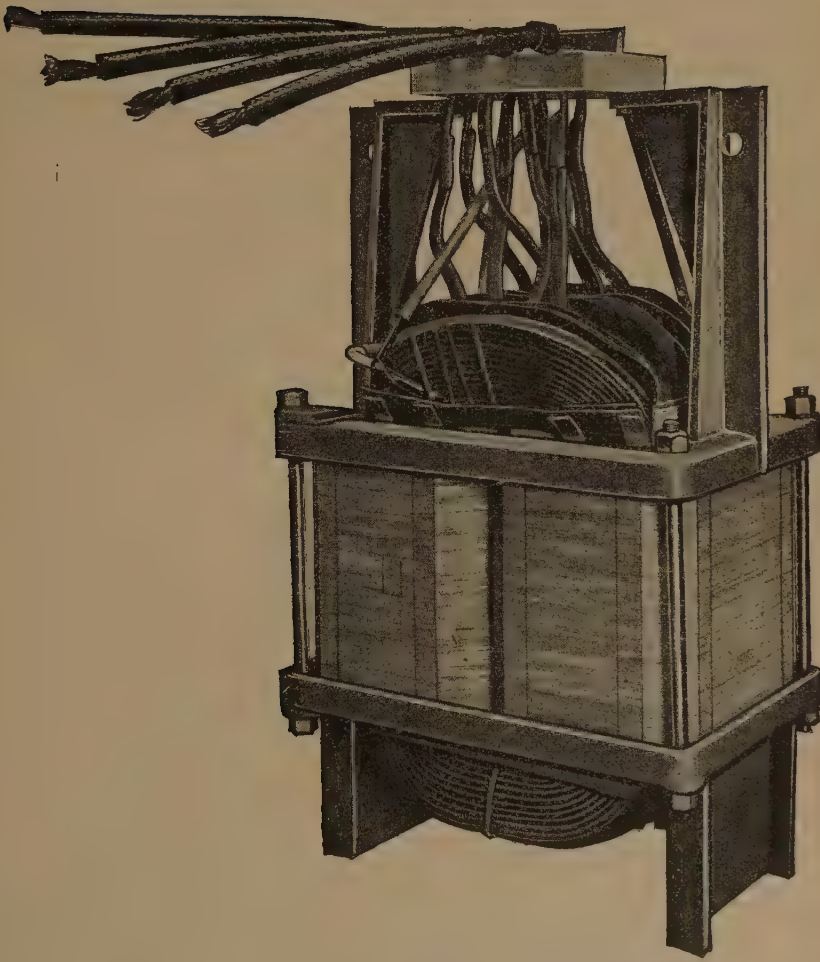

FIG. 39. Shell Type Transformer.

In the core type we have a long magnetic circuit and a short copper circuit. In the shell type a short magnetic and a long copper circuit. The iron laminations in the core type are rectangular in shape, and hence there is no waste of material. In order to get the advantage of a circular form for the copper

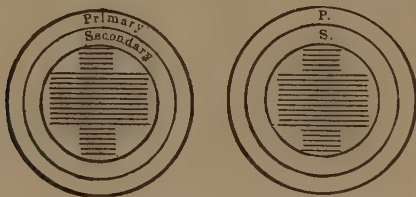


FIG. 40. Core Type Transformer.

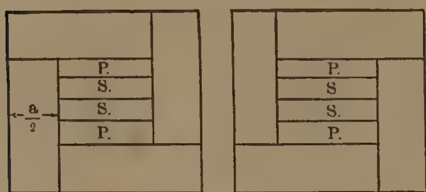


FIG. 41. Shell Type Transformer.

and still have the coils fit the iron as closely as possible, one manufacturer gives the core a section as shown in Fig. 40.

For the shell type a rectangular punching may be used and the core built up as shown in Fig. 41. For large transformers this construction is nearly always used. To reduce the number of air gaps and still

not have much waste material one manufacturer uses a punching as shown in Fig. 42. Another manufacturer has accomplished the same thing by

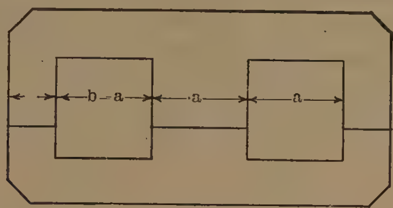


FIG. 42. Transformer Punchings.

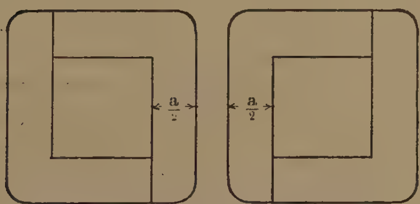


FIG. 43. Transformer Punchings.

using a punching as shown in Fig. 43. In large transformers the core is nearly always divided in the middle as shown in Fig. 41, in order to increase the radiating surface.

**64.** The core type has the advantage of low cost of construction, good insulation and large radiating surface, the coils being especially well ventilated. The core type has the disadvantage of large magnetic

leakage, especially when built for high voltages, on account of the long magnetic circuit. The long magnetic circuit necessitates a large number of turns in order to keep the open circuit current from being excessive. A large number of turns and a long magnetic circuit are disadvantageous when the transformer is to regulate well for inductive and capacity loads. For large capacities, or when built for high voltage, the shell type is generally used.

The shell type is generally more expensive to construct, is more difficult to insulate and has smaller radiating surface, since the construction is more compact. It has the advantage of a short magnetic circuit and small number of turns. The advantages and disadvantages of the two types will be made clear by following through the design of a transformer of each type.

A distinct difference exists between European and American practice with regard to transformers to be used on polyphase circuits. In Europe three-phase transformers are very generally used. For three-phase circuits such transformers have three legs united by yokes common to the three, one phase being wound on each leg (Fig. 44). For two-phase work the same magnetic structure may be used, the middle core being unwound and having a section equal to the $\sqrt{2}$ times either of the others. The advantages of this construction, especially for three-phase, are economy of material and compactness. These small advantages appear to be offset by

constructional difficulties, and by the fact that a breakdown of one leg throws the whole unit out of service. In America the practice of using single

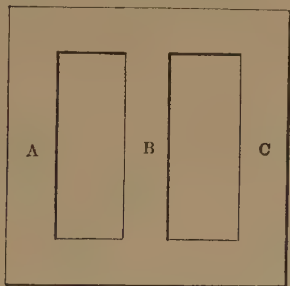


FIG. 44. Three Phase Transformer Core.

transformers grouped in different ways for three-phase work is quite general, and such practice seems to commend itself because of the advantages it presents.

## DESIGN OF TRANSFORMERS.

**65.** We have as the fundamental equation in transformer design:

$$E=\sqrt{2}\,\pi SABf/10^8. \qquad (29)$$

$A$ is the area of cross-section of core.

At a frequency of 60 cycles* per second, this becomes

$$\frac{E\times 3.75\times 10^5}{B}=SA. \qquad (30)$$

*Commercial frequencies vary from about 15 to 150.

SHELL TYPE.

Let us design a transformer of the shell type. In order to get the simplest results, it is well to express all linear dimensions in terms of one dimension of the transformer. We shall refer all dimensions to the length $a$, Fig. 45 ($a$ is in centimeters). The width

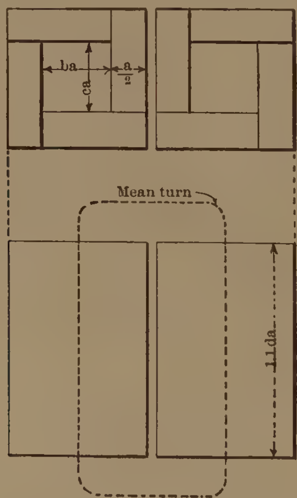


FIG. 45. Shell Type Transformer.

of the opening for the copper is $ba$. $b$ is usually about unity. The depth of the opening for the winding space is $ca$. $c$ is usually between one and two. The length of the core, not allowing for iron insulation, is $da$, and allowing 10% for insulation it is $1.1da$. $d$ usually varies from $1^1/_2$ to 4. Let us assume that the ratio of the area of the winding

space to the total cross-section of copper in primary and secondary is $r$. $r$ for 2,000 volt shell type transformers is about 1.5 to 2. For higher voltages $r$ must increase. The number of turns on the winding, having an e.m.f. $E$, is $S$. The area of cross-section of one winding is $a^1S$, $a^1$ being the cross-section of one wire in square centimeters. Since the current varies inversely with the number of turns, the total cross-section of both windings, assuming the same density in each winding, will be $2a^1S$. The ratio of the area of the winding space to space occupied by copper is $r$, therefore,

$$2a^1S = bca^2/r, \text{ giving} \tag{31a}$$

$$a^1 = bca^2/2rS. \tag{31b}$$

But we have from (30), putting $da^2$ for $A$, $Sda^2B = E \times 3.75 \times 10^5$; therefore,

$$a^1 = \frac{bcda^4B}{2rE\ 3.75 \times 10^5} \tag{31c}$$

If we assume a density of 150 amperes per square centimeter (this corresponds to 1,300* circular mils per ampere), we get by multiplying the above equation by 150 $E$,

$$\text{Capacity in watts} = \frac{bcda^4B}{5r\ 10^3},$$

$$\text{Capacity in K. W.} = \frac{bcda^4B}{5r\ 10^6}. \tag{32}$$

*This density is quite common.

**66.** The volume of iron, not allowing for any corners that may be cut off, is

$$V = 2da^3\ (c + b + 1).$$

$$\text{Weight of iron in pounds} = \frac{2 \times 7.7 da^3\ (c + b + 1)}{454}$$

(Density of iron assumed 7.7.)

Divide this equation by (32) and we get the weight of iron per K. W.

Weight of iron per K. W.

$$= \frac{17 \times 10^4 r\ (c + b + 1)}{bcaB}. \qquad (33)$$

**67.** If $kB^{1.6}$ gives the iron loss in watts per pound of iron at 60 cycles per second, then the percentage iron loss will be

Percentage iron loss

$$= \frac{\text{Wt. of iron per K.W.} \times kB^{1.6} \times 100.}{1000} \qquad (34)$$

The value of $kB^{1.6}$ may be taken directly from a curve as given in Fig. 34 for any particular value of $B$. The percentage of full load current on open circuit to supply the hysteresis loss will also be given by equation (34).

**68.** The mean length of a copper turn may be taken as $2.2da+2a+8\ (ba/2)$ (this will allow sufficient length for the division of the core at the middle as shown in the figure). The volume of copper is equal to this mean length multiplied by the cross-section, giving

$$V=2a^1S\ (2.2da+2a+4ba).$$

Or, substituting for $2a^1S$ the value given in (31) we get

$$\text{Wt. of copper}=\frac{2\times 8.9bca^3}{454r}(1.1d+2b+1).$$

Dividing this equation by (32) we get

$$\text{Wt. of copper per K. W.}=\frac{19.6\times 10^4}{daB}(1.1d+2b+1). \qquad (35)$$

**69.** The percentage $IR$ drop may be determined as follows: The length of one winding is

$$S\ (2.2da+4ba+2a).$$

Replacing $S$ by its value in terms of $a$ we get

$$\text{Length of copper}=\frac{2E3.75\times 10^5}{daB30.5}(1.1d+2b+1)\ \text{(in ft.)}$$

Since 1,300 circular mils are allowed to one ampere,

the $IR$ drop in one winding will be, assuming the resistance per mil foot to be 11,

$$IR = \frac{2E 3.75 \times 10^5 \; (1.1d + 2b + 1) \; 11}{daB \times 30.5 \times 1300}$$

And the percentage $IR$ drop in both windings will be this value multiplied by $2 \times 100/E$, giving

$$\text{Percentage } IR \text{ drop} = \frac{42 \times 10^3}{daB}(1.1d + 2b + 1) \qquad (36a)$$

$$= .213 \text{ (wt. of copper per K. W.)} \qquad (36b)$$

**70.** The mean length of the magnetic circuit is

$$l = 2ca + 2ba + 2a.$$

The magnetizing current in virtual amperes is

$$i_m = .565 \; Bl/\mu S.$$

In place of $l$ put its value from above, and in place of $S$ its value from (31), and we get

$$i_m = \frac{.565B \; (c + b + 1) \; 4ra^1}{\mu bca}$$

Multiply and divide the right hand member by 150, and substitute for $150a^1$ the full load current of the transformer. Then the magnetizing current in percentage of full load current will be given by the equation:

Percentage magnetizing current

$$=\frac{1.5(c+b+1)rB}{bca\quad\mu}. \tag{37}$$

The percentage open circuit current may be obtained by taking the square root of the sum of the squares of (34) and (37).

**71.** The copper radiating surface without spreading the coils, is

$$\frac{\pi a^2 [(1+b)^2+c\,(1+b)]}{2} \tag{38a}$$

The iron radiating surface, assuming the core divided as in Fig. 41, is

$$2a^2\,(2d+2cd+2bd+2b+c+2) \tag{38b}$$

**72.** Collecting the equations which are important, we get

$$Sda^2B=E3.75\times10^5. \tag{30}$$

$$\text{Capacity in K. W.}=bcda^4B/5r10^6 \tag{32}$$

Wt. of iron per K. W.

$$=\frac{17\times10^4r\,(c+b+1)}{bcaB}\ \text{(in lbs.)} \tag{33}$$

Percentage iron loss

$$=\frac{\text{wt. of iron per K.W.}\times kB^{1.6}}{10} \tag{34}$$

Wt. of copper per K. W.

$$=\frac{19.6\times 10^4 \times (1.1d+2b+1)}{daB} \text{(in lbs.)} \tag{35}$$

$$\text{Percentage } IR \text{ drop} = \frac{42\times 10^3\,(1.1d+2b+1)}{daB} \tag{36a}$$

$$= .213 \times \text{wt. of copper per K. W.} \tag{36b}$$

Percentage magnetizing current

$$=\frac{1.5\;(c+b+1)rB}{bca\qquad\mu} \tag{37}$$

For any other frequency multiply right hand member of (32) by $f/60$, and (30), (33), (35) and (36) by $60/f$. For any other circular mils per ampere multiply (32) and (36$a$) by $\dfrac{1300}{\text{c.m. per ampere,}}$ and (33), (34), (35) and (37) by $\dfrac{\text{c.m. per ampere}}{1300}$.

**73.** In practice, one or more of the values $b$, $c$, $d$, $a$, or $B$, may be fixed. The stamping may be given and it may be required to find the length of the core to give the required capacity. Or, the problem may be to find what values of $b$, $c$, and $d$, will give us the best transformer for the given conditions. Let us assume $b=1$, $c=1.5$, $d=3$, $B=6{,}000$, and $r=1.5$,

and make a calculation for a 5 K. W. transformer. Substituting the values given in (32) we get $a = 6.13$ centimeters. Substituting this value in (30) we get for the secondary turns, if the secondary voltage is 110, $S = 62$. Substituting in the other equations, we get

Wt. of iron per K. W. = 16.2 lbs.
Percentage iron loss = 1.13.
Wt. of copper per K. W. = 11 lbs.
Percentage $IR$ drop = 2.33.
Percentage magnetizing current = .94.
Percentage open circuit current

$$= \sqrt{(1.13)^2 + (.94)^2} = 1.6.$$

Power factor on open circuit = 1.13/1.6 = .7.
Radiating surface = 400 square inches.
Total loss at full load in watts

$$= 5{,}000\ (2.33 + 1.13) = 173 \text{ watts}.$$

Watts lost per square inch of radiating surface = .4.

The approximate rise in temperature may be determined from the curves in Fig. 46. As seen from the curves the temperature rise would be too high, and the losses would have to be reduced or the radiating surface increased.

It is easily seen by inspection of the formulæ 30 to 37 what must be done to change these results in any desired way. If, for instance, the percentage $IR$ drop is too large, it is evident from a glance at (36$a$) that the only thing that can be done to appreciably

affect the result (since $a$ and $B$ cannot be increased much more) is to increase the circular mils per ampere.

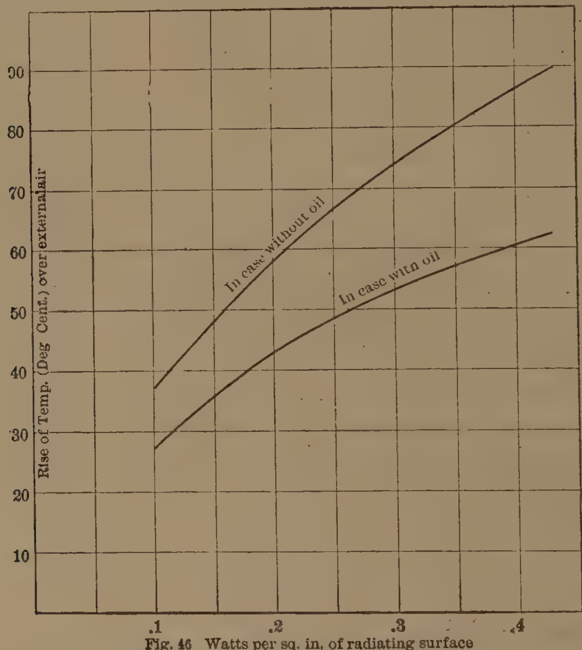


FIG. 46. Watts per Square Inch Radiating Surface.
(From data by Mr. Gisbert Kapp.)

**74.** The capacity increases as the fourth power of the linear dimensions, the losses as the third power, and the radiating surface as the second power. For large transformers, therefore, it becomes necessary to provide more efficient means of getting rid of the

heat or to reduce the losses. Suppose we see the effect of doubling the value $a$, leaving everything else the same. The capacity, losses, etc., of the 5 K. W. transformer we will call unity.

| | STANDARD (5 K. W.) DIMENSION $a$. | NEW DESIGN DIMENSION $2a$ |
|---|---|---|
| Turns | 1 | 1 |
| K. W. | 1 | 16 |
| Wt. of iron per K. W. | 1 | $1/2$ |
| Wt. of copper per K. W. | 1 | $1/2$ |
| Per cent. iron loss | 1 | $1/2$ |
| Per cent. copper loss | 1 | $1/2$ |
| Per cent. open circuit current | 1 | $1/2$ |
| $\frac{\text{Watt loss}}{\text{Radiating surface}}$ | 1 | $1/2$ |

The large transformer has half the percentage losses of the smaller but the watt loss per square inch of radiating surface is also reduced one-half. In the large transformers, if these values are to obtain, more efficient means of cooling must be provided, unless in the smaller the rise of the temperature was considerably below the limit.

Small transformers are usually air cooled. Larger transformers are submerged in oil. In the very large sizes, coils carrying running water are put into the containing tanks to keep the temperature down to reasonable limits.

In the design of large transformers, care must be taken to subdivide the copper in order to prevent

eddy current loss. The copper area must be divided and the parts insulated from each other. The failure to divide the copper may result in an eddy current loss within the conductor at no load so great as to burn the insulation.

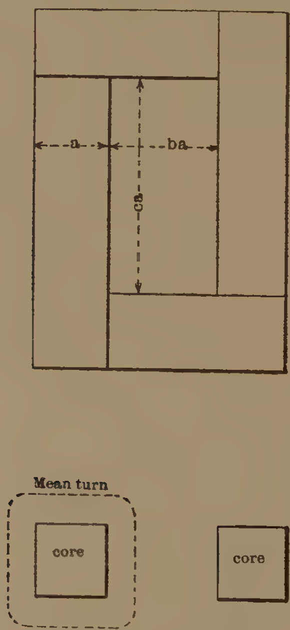


FIG. 47. Core Type.

CORE TYPE.

75. We may treat the core type in a similar way. Referring to Fig. 47, *a* is the width of the lamination,

We will assume a square section for the core. If the core is not square, the formulæ derived will apply with little error if we determine for the given cross-section the equivalent square and use the equivalent value of $a$ thus found. The width of opening is $ba$ and the length $ca$. $b$ is usually between 1 and 2, and $c$ between 3 and 5. The ratio of winding space to copper is greater in the core type than in the shell type. $r$ may be between 2 and 3 for 2,000 volt transformers.

We have for this case $BSa^2 = E \times 3.75 \times 10^5$. (38)
As before we have

$$2a'S = bca^2/r, \text{ giving}$$
$$\text{Capacity in K. W.} = bca^4B/5r10^6. \quad (39)$$

The volume of iron is $2a^3\ (b+c+2)$, and hence

$$\text{Wt. of iron per K. W.} = \frac{17 \times 10^4 r\ (b+c+2)}{bcaB}. \quad (40)$$

$$\text{Percentage iron loss} = \frac{\text{wt. per K. W.} \times kB^{1.6}}{10}. \quad (41)$$

The mean length of a copper turn may be taken as $4(a+ba/2)$, and the volume of copper becomes $2a'S \times 4\ (a+ba/2)$. Substituting $bca^2/r$ for $2a'S$, we get

$$\text{Vol. of copper} = \frac{2bca^3}{r}(2+b).$$

Hence,

$$\text{Wt. of copper per K. W.} = \frac{19.6 \times 10^4\ (2+b)}{aB}. \quad (42)$$

$$\text{Percentage } IR \text{ drop} = \frac{42 \times 10^3\ (2+b)}{aB} \qquad (43a)$$

$$= 0.212 \times \text{wt. of copper per K. W.} \qquad (43b)$$

The length of the magnetic circuit is $2a \times (c+b+2)$.

The percentage magnetizing current becomes,

$$\left.\begin{matrix}\text{Percentage magnetizing}\\ \text{current}\end{matrix}\right\} = \frac{1.5\ (c+b+2)rB}{bca\qquad \mu} \qquad (44)$$

The copper radiating surface is

$$4a^2\ (2c+2b+b^2+2bc). \qquad (45a)$$

The iron radiating surface is $2a^2\ (3b+8)$ (45b)

**76.** Collecting formulæ,

$$BSa^2 = E \times 3.75 \times 10^5. \qquad (38)$$

$$\text{Capacity in K. W.} = bca^4B/5r10^6. \qquad (39)$$

$$\text{Wt. of iron per K. W.} = \frac{17 \times 10^4 r(b+c+2)}{bcaB} \qquad (40)$$

Percentage iron loss

$$= \frac{\text{wt. of iron per K. W.} \times kB^{1.6}}{10} \qquad (41)$$

$$\text{Wt. of copper per K.W.} = \frac{19.6 \times 10^4\ (2+b)}{aB} \qquad (42)$$

$$\text{Percentage } IR \text{ drop} = \frac{42 \times 10^3\ (2+b)}{aB} \qquad (43a)$$

$$(\text{at } 70^\circ \text{ F.}) = 0.212 \times \text{wt. of copper per K.W.} \qquad (43b)$$

Percentage magnetizing current

$$= \frac{1.5\ (c+b+2)rB}{bca \qquad \mu} \tag{44}$$

For any other frequency, multiply right hand member (39) by $f/60$, and (38), (40), (42) and (43) by $60/f$. For any other circular mils per ampere, multiply (39) and (43$a$) by $\frac{1300}{\text{c.m. per ampere}}$, and (40), (41), (42) and (44) by $\frac{\text{c.m. per ampere}}{1300}$.

**77.** Let us calculate a 5 K. W. core type transformer. Take $b=2$, $c=3$, $r=2$, and $B=6{,}000$. We get from (39) $a=6.14$. This gives $S$ about 180 for 110 volts, which is too large. Increase $B$ to 9,000, and decrease $b$ to 1.5; this gives us $a=$ about 6, and $S=120$. This is large for $S$, but will do. Substituting these values in (40), (41), etc., we get

Wt. of iron per K. W. = 9.2 lbs.
Percentage iron loss = 1.17.
Wt. of copper per K. W. = 12.5 lbs.
Percentage $IR$ loss = 2.5.
Percentage magnetizing current = 1.43.
Percentage open circuit current

$$= \sqrt{(1.17)^2 + (1.43)^2} = 1.9.$$

Power factor = 1.17/1.9 = .62.
Watts loss at full load = 5,000 (.0117 + .025) = 184.
Watts loss per square inch of radiating surface = .36.

**78.** Comparing the two types for the dimensions chosen we get:

| | CORE. | SHELL. |
|---|---|---|
| Wt. of iron ............... | 46£ | 81£ |
| Wt. of copper ............. | 63£ | 55£ |
| Total wt................... | 109£ | 136£ |
| Per cent. iron loss ......... | 1.17 | 1.13 |
| Per cent. copper loss ....... | 2.5 | 2.33 |
| Per cent. open circuit current | 1.9 | 1.6 |
| Power factor. .............. | .62 | .7 |
| Total loss at full load ...... | 3.67% | 3.46% |
| $\frac{\text{Watt loss}}{\text{Radiating surface}}$ ........... | .36 | .4 |

The core type is generally characterized by small weight of iron and large weight of copper, a large number of turns and large open circuit current. The increased cost of copper in the core type is off-set by low cost of insulating material and labor of assembling as compared to the shell type.

**79. Design of Induction Coils.**—Induction coils of large capacity are beginning to be used for neutralizing the charging current of transmission lines. For a line 150 miles long, and operating at 60,000 volts, at 60 p.p.s., the charging current is about 31 amperes. This means that the apparent energy put into the line at no load is about 3,000 K. W. It is desirable that this be reduced, and it may be accomplished by the use of induction coils. It will generally be economical to neutralize about 50%

of the charging current—more or less depending on the particular conditions. Whether or not coils are to be used in any particular case depends *on the ratio of the load current to the charging current.*

Recently there have been installed, by the Bay Counties Power Co., of California, on the recommendation of the author, three coils having an aggregate capacity of 2,500 kilo-volt-amperes at 60,000 volts, the line potential for which the coils were calculated.

**80.** The coils may be constructed like core type transformers having no secondary winding. There must, however, be a small air-gap in the magnetic circuit to increase the lag angle of the current taken by the coil. Equation (29) applies to coils as well as to transformers. The other formulæ developed for the transformer apply also excepting the one for the magnetizing current. The magnetizing current of the coils is the total current they are to take from the line. The iron part of the magnetic reluctance may be neglected in comparison with the air-gap reluctance; the current taken by the coils may therefore be written

$$I = \frac{Bl}{4\pi S\sqrt{2}},$$

in which $l$ is the length of the air-gap. After choosing $B$ we see that $S$ depends on the length of the air-gap. In order to get an economical design $S$ must not be too small. For small values of $l$ the cost of materials varies practically inversely as $l$.

**81.** On account of the large magnetic potential which would exist across the air-gap, if only one or two gaps were put in the magnetic circuit, a large stray field would be produced which would result in excessive eddy currents in the iron laminations. To reduce the eddy currents a large number of air-gaps should be put in the magnetic circuit.

**82. Compensators.**—For certain purposes transformers are built with a single winding tapped at different points, as shown in Fig. 48. As a transformer, this method of construction is very inferior in every way to that where two separate windings

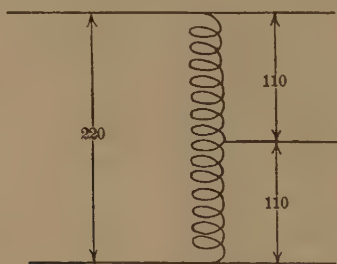


FIG. 48. Compensator.

are used. Compensators, or economy coils, as they are called, are very often used to reduce the voltage for arc lighting on constant potential mains. Such transformers, sometimes called auto-transformers, are also used to reduce the voltage in starting certain types of induction motors.

# CHAPTER VIII.

## CIRCUIT REGULATORS, SERIES TRANSFORMERS, CONSTANT CURRENT TRANSFORMERS, COMPENSATING VOLT-METER.

**83. Circuit Regulators.**—In large stations where there are several circuits supplied from the same generator, or from the same set of bus-bars, it becomes necessary, in order to keep all the lamps at the same brilliancy, to provide some means of adjusting the potential of each circuit independently. Suppose a single-phase generator feeds several circuits, the circuits being different in length and having loads differing in character and amount. Without some means of changing the pressure of any circuit independently it will, obviously, be impossible to keep constant potential at the lamps for all the possible changes of load in the different circuits. This trouble is remedied by a separate circuit regulating device.

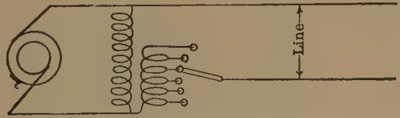


FIG. 49. Stillwell Regulator.

**84.** Fig. 49 shows diagrammatically the Stillwell regulator, made by the Westinghouse Company, for

adjusting the pressure of different circuits independently. Evidently connections may be made to

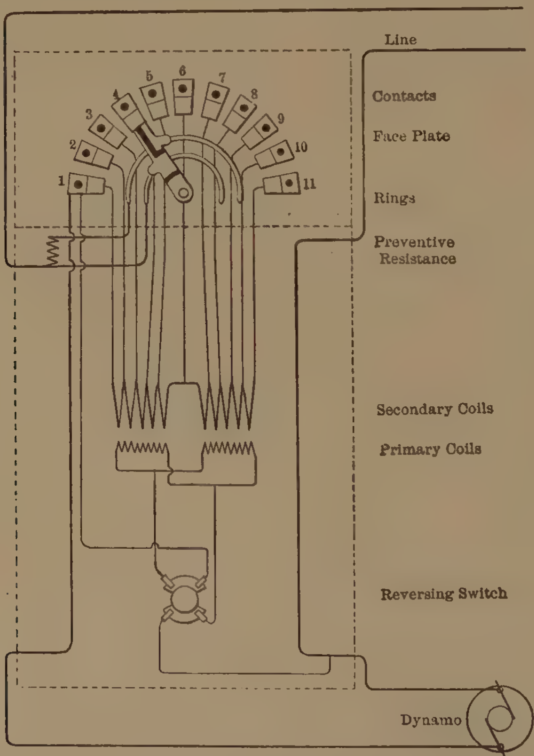


FIG. 50. Internal Connections of Stillwell Regulator.

raise or depress the potential of the line. Fig. 50 shows the internal connections of this regulator.

**85.** The power transformed in the regulating transformer is usually not more than 10% of the power delivered to the circuit and since the regulating transformer has an efficiency of at least 95%, the regulating device does its work with no appreciable decrease in the efficiency of the station.

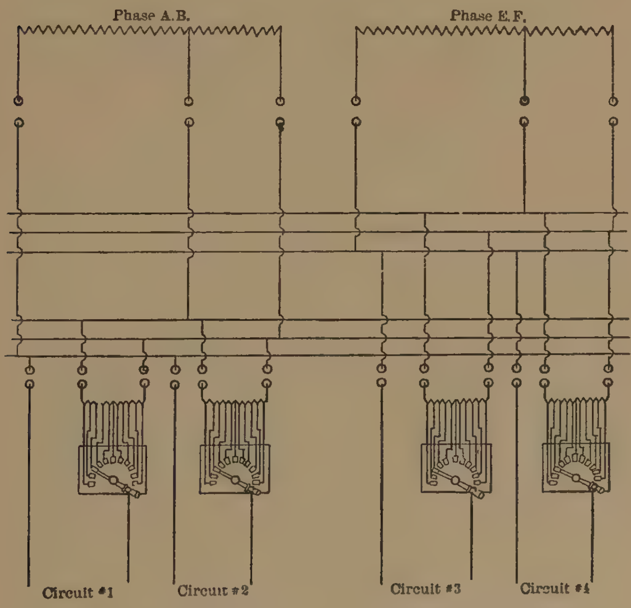


FIG. 51. Stanley Co.'s Method of Regulating Potential.

**86.** Fig. 51 shows the Stanley Company's scheme for regulating the pressure of the circuits. Three leads are brought to the switch-board from each

phase of the generator. Across two of these a regulating transformer is connected as shown.

**87.** Fig. 52 shows one type of regulating device made by the General Electric Company. The primary and secondary coils are stationary and at right

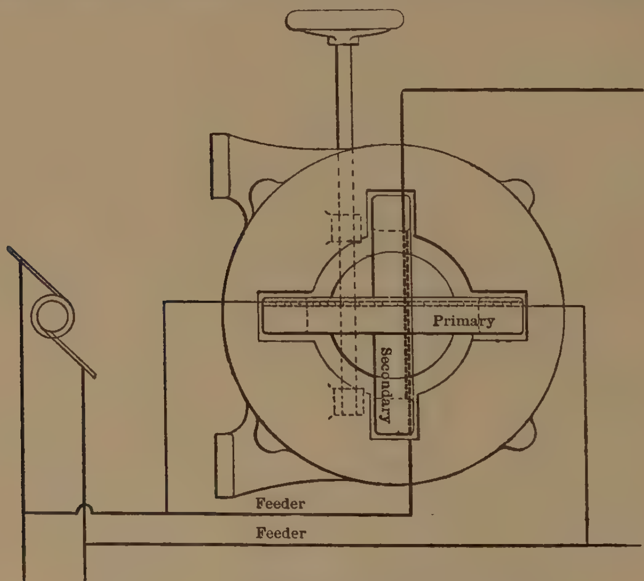


FIG. 52. Connections for Feeder Regulator.

angles to each other, the core being movable. By rotating the core the voltage may be raised or lowered without any change of connections. This type of regulator, on account of the air-gap between primary and secondary, necessarily introduces more

inductance in the line than those previously described. This company also manufactures a type of regulator similar to the Stillwell regulator. A type of regulator for three-phase circuits is shown in Fig. 53.

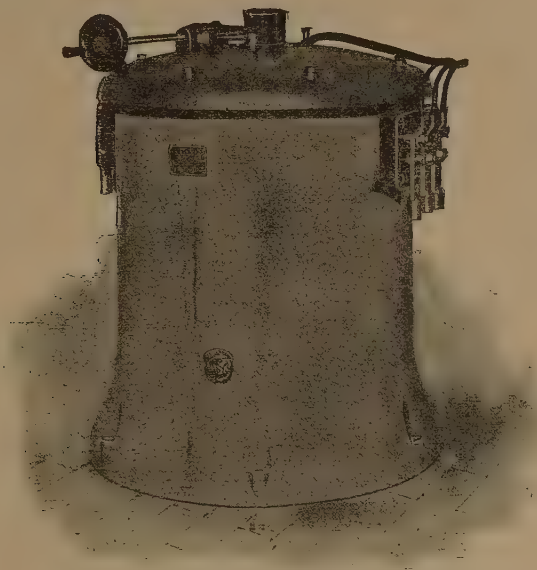

FIG. 53. Three-Phase Potential Regulator.

Referring to Fig. 54, which represents graphically the voltage of a single phase of the regulator, $e\ o$ = Generator voltage or the e.m.f. impressed on the primary; $a\ o$ = e.m.f. generated in the secondary coils, and is constant with constant generator e.m.f.;

$b'\ a'$ = secondary e.m.f. in phase with the generator e.m.f.; $e'\ a'$ = line e.m.f. or resultant of the generator e.m.f. and the secondary e.m.f.

The construction of the regulator is such that the secondary voltage $o\ a$ is made to assume any desired phase position relative to the primary e.m.f. as $o\ f$, $o\ b$, $o\ c$, etc.

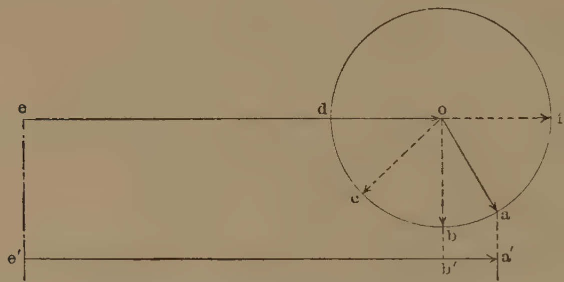


FIG. 54. Showing Theory of Regulator.

When its phase relation is as represented by $o\ f$, which is the position when the north poles and the south poles of the primary and secondary windings are opposite, the secondary voltage is in phase with the primary voltage and is added directly to that of the generator. The regulator is then said to be in the position of maximum "boost," and by rotating the armature with reference to the fields, the phase relation can be changed to any extent between this and directly opposed voltages. When the voltage of the secondary is directly opposed to that of the primary or generator, its phase relation is as represented by $o\ d$ in the diagram, while $o\ b$ represents

the phase relation of the secondary when in the neutral position.

In order to avoid the trouble experienced in adjusting the voltages when each phase is controlled independently, the polyphase potential regulators are arranged to change the voltage in all phases simultaneously.

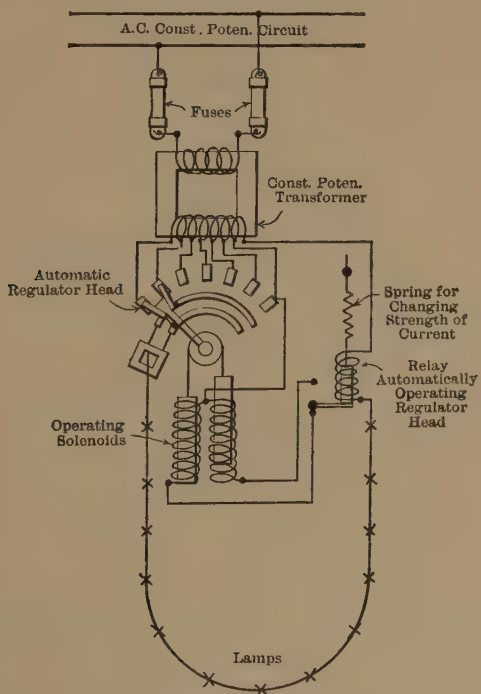


FIG. 55. Alternating System of Arc Lighting.

**88. The Constant Current Transformer.**—Instead of constant voltage, we sometimes wish to produce constant current in the secondary with constant potential on the primary. We may do this by varying automatically (1) the number of turns on the secondary, (2) the magnetic reluctance between primary and secondary, (3) the impedance in the primary or secondary.

The first method, shown in Fig. 55, is used commercially on a small scale in this country and has a

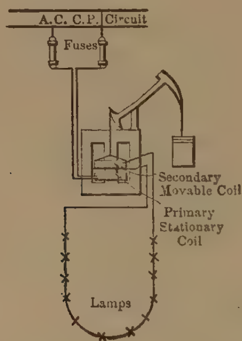


FIG. 56. Constant Current Transformer.

great many points in its favor. The second method, shown in Fig. 56, is used in the General Electric Company's floating coil transformer. The secondary is made movable with respect to the primary, an increase in the current causing the secondary to be

repelled further from the primary on account of the repulsion of the primary and secondary coils. Being further away from the primary, there is less induced pressure and hence, a tendency to keep constant current. In the third method, shown in Fig. 57, circuits to the lamps are run directly from constant potential transformers. In each circuit, however, is connected a choke coil with variable self-induction.

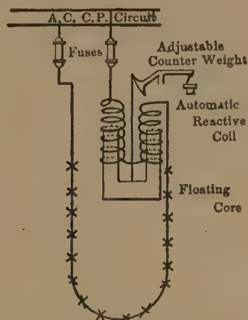


FIG. 57. Alternating System of Arc Lighting.

The motion of the coil with respect to the core causes more or less e.m.f. to be consumed by the coil.

The second and third methods introduce poor power factors on the line on account of the poor magnetic circuit in one case and the introduction of a choke coil in the other. The first method is not open to this objection.

**89. The Series Transformer.**—Very often we wish to measure the current or power in the high potential side of alternating current lines, but for safety it is not advisable to connect an ammeter or watt-meter directly in series with the line. In such cases we can get rid of the metallic connection to the line by the use of a series transformer, Fig. 58. One winding is connected in series with the line and the other in series with the ammeter or watt-meter. It can be readily seen that, because the resistance of the transformer is not negligible in comparison with the im-

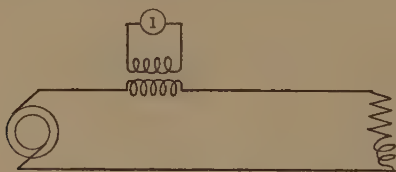


FIG. 58. Series Transformer.

pedance and on account of the impedance introduced by the ammeter or watt-meter, the current of the secondary in this case depends (1) on the ratio of turns, (2) on the resistance and impedance of the transformer and the instruments connected in series with it, (3) on the permeability of the magnetic circuit of the transformer and instrument. To be accurate, therefore, an ammeter or indicating watt-meter must be calibrated with the series transformer with which it is to be used, and the calibration must be made at the frequency of the circuit in which it is to be connected. On

account of the change in permeability for different values of current there will not be a straight line relation between the primary and secondary currents and, therefore, recording watt-meters will not record accurately throughout their range when used with series transformers, although, by a proper design, the error can be made small.

**90. Compensating Volt-meter.**—In operating a station it is important to keep the pressure at the lamps nearly constant. In order to do this, some means must be provided for determining the pressure at the centre of distribution. Pressure wires may be run back to the station and the voltage deter-

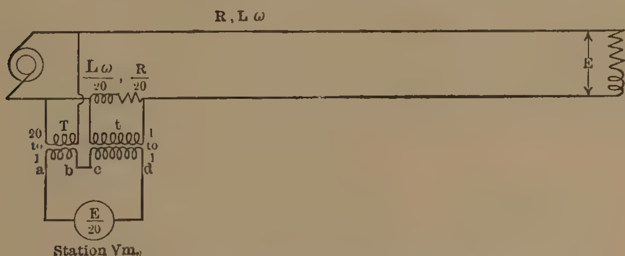


FIG. 59. Compensated Volt Meter Connections.

mined in this way. This, however, is expensive. In order to avoid the use of pressure wires, Mr. Mershon devised a compensator by means of which the station volt-meter may be made to read the pressure at the centre of distribution, approximately, for any line current or any power-factor of the load.

Fig. 59 shows a diagram by which we may explain

the theory of the compensated volt-meter. The resistance of the line is $R$ and the reactance $L\omega$. The pressure at the centre of distribution is $E$. A current, $I$, flowing over the line consumes pressure due to the resistance equal to $IR$, and a pressure due to the reactance of the line equal to $L\omega I$. At the generating station we have a potential transformer, $T$, with a ratio of transformation of, say, 20 to 1. In series with the line at the station is placed a coil, the reactance of which is equal to $L\omega/20$ and the resistance equal to $R/20$. It is clear, now, that the pressure consumed across the series coil will be 1/20 of the pressure consumed over the line, the phase of the pressure being equal to the phase of the pressure consumed over the line. Across the series coil place a transformer having a ratio of 1 to 1. If we connect the secondaries in series the volt-meter may be made to read the voltage $E/20$. Care must be taken to connect *ab* and *cd* in the right way. This may be done by noting the voltage across *ab* and *ad* when a lighting or motor load is on the line. The volt-meter should read less when connected across *ad* than when connected across *ab*. If the transformer $t$ has a ratio of 1 to 10, the series coil may be made smaller, the reactance being $L\omega/200$ and the resistance $R/200$. Sometimes a compensator may be improvised by connecting a step-up transformer, $t$, across a certain length of the bus-bars in the station and connecting this in series with a transformer, $T$, connected across the bus-bars at the station.

# CHAPTER IX.

## TRANSFORMER CONNECTIONS.

**91.** The main advantage of the alternating current over continuous for a general system of distribution is due to the facility with which the pressures may

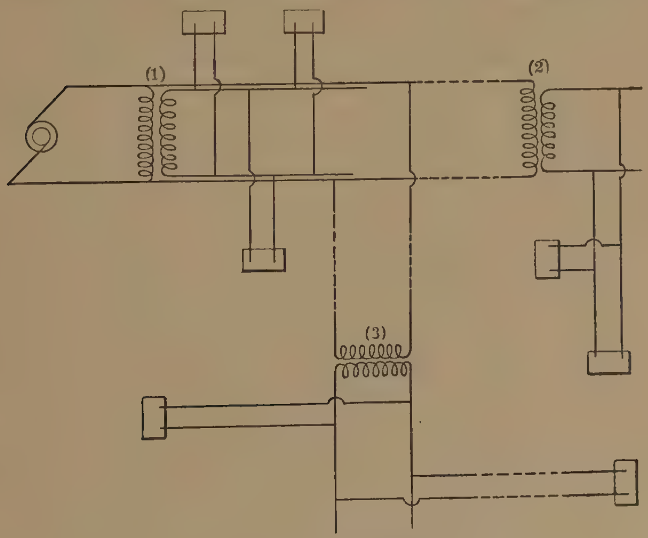


FIG. 60. Single Phase Distributing System.

be changed at any desired point by the use of transformers. Were it not for the transformer, our long distance transmission systems would not be in

existence. In this chapter some of the common connections of transformers will be given.

Fig. 60 shows diagrammatically the system of distribution now commonly used. The territory to be covered is divided into districts, and near the centre of distribution of each district is placed one or more transformers of sufficient capacity to carry the load, a secondary distributing system being run to the individual consumers. Each bank of transformers is generally fed over a separate line from the station, so that regulators may be placed in the line and the pressure for each bank of transformers regulated independently. In districts which are near together, the secondary systems are inter-connected. Such a system necessarily uses more copper than the old system, where each consumer was supplied with a separate transformer. The gain in efficiency and decreased cost of transformers, however, more than counterbalances the extra cost of copper.

92. Nearly all manufacturers now make trans-

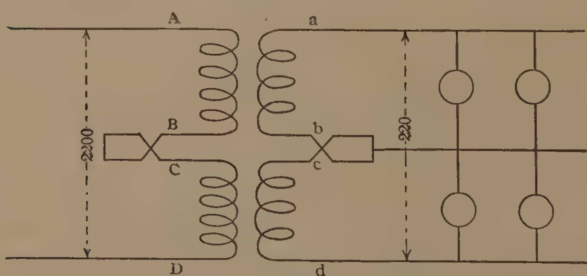


FIG. 61. Primaries in Series, Secondaries in Series.

formers with two primary and two secondary coils. The standard primary voltages are about 2,200 and 1,100 with the secondary 220 or 110. Figs. 61 to

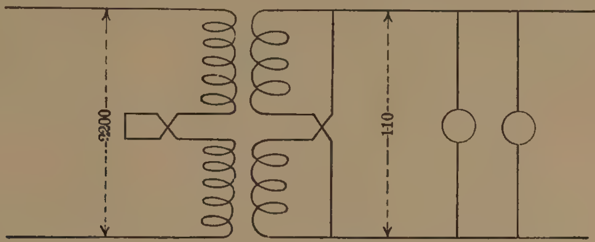


FIG. 62. Primaries in Series, Secondaries in Parallel.

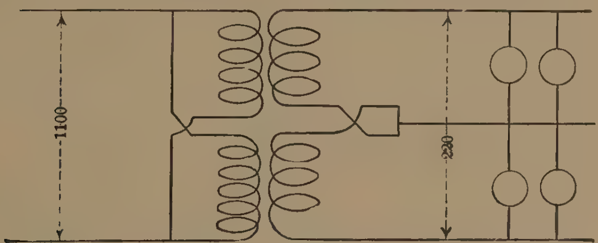


FIG. 63. Primaries in Parallel, Secondaries in Series.

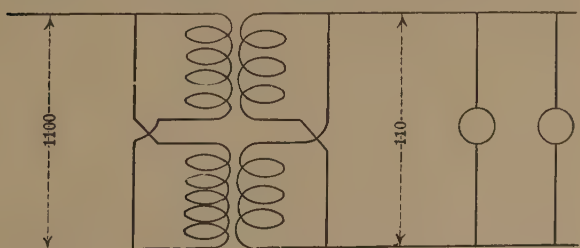


FIG. 64. Primaries in Parallel, Secondaries in Parallel.

64 show the different ways of connecting a transformer having two primary and two secondary coils.

**93.** Where lights are supplied on the three-wire system, it becomes necessary to guard against any unbalancing of voltage due to unequal loading of the two sides of the system. In well constructed shell-type transformers practically no unbalancing of pressures takes place. In the core type, however,

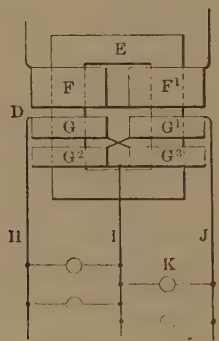


FIG. 65*a*. G. E. Co.'s Method of Preventing Unbalancing of Pressures.

if care is not taken so that a load on either side of the three-wire system produces the same magnetomotive force on each leg of the core, the secondary pressures will become unbalanced, if the two sides of the system are loaded unequally.

Fig. 65*a* shows the General Electric Company's method of preventing unbalancing. Fig. 65*b* shows the method adopted by the General Electric Company of Berlin for single-phase transformers, and 65*c* shows the method applied to three-phase transformers.

The lower figure in each case gives the method adopted to prevent the unbalancing which would take place if connections were made as in the upper figure.

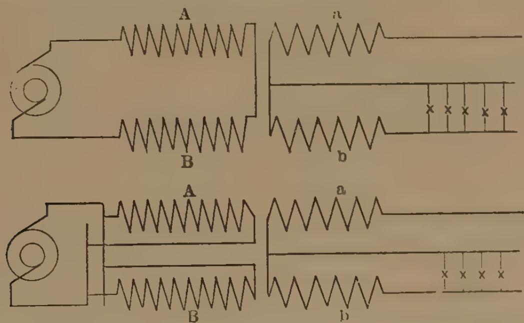


FIG. 65*b*. Transformer Connections for Balanced Pressures.

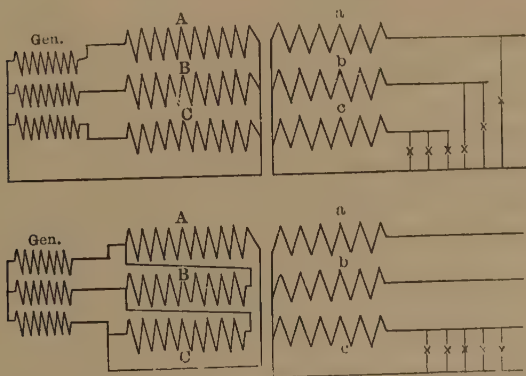


FIG. 65*c*. Transformer Connections for Balanced Pressures.

As shown, each primary coil is wound for the full line voltage and the coils are then connected in parallel.

**94. Three-Phase Connections.**—Single-phase transformers may be used on three-phase systems in several ways. The most common connections are shown in Figs. 66 and 67. Fig. 66 shows the

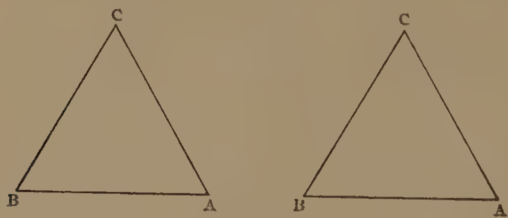


FIG. 66. Δ to Δ Connection.

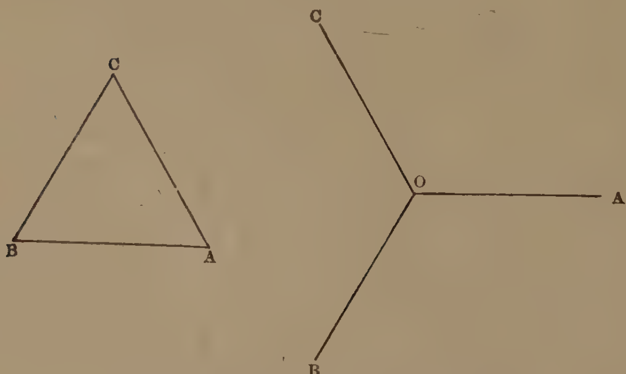


FIG. 67. Δ to Star, or Y, Connection.

"Delta" (Δ) to "Delta" (Δ) connection. The three transformers are connected up in a ring as shown and the three line wires then connected to the points

$A, B, C$. Both windings may be connected "Delta" as shown in Fig. 66, or one may be connected "Delta" and the other "Star," or $Y$, as shown in Fig. 67, or both windings may be connected "Star." In order to balance the loads one side of the trans-

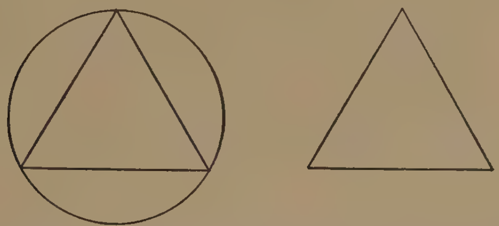

FIG. 68. Δ Connection for Three-Phase Converter.

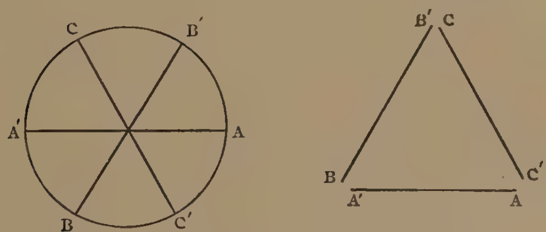


FIG. 69. Six-Phase Converter Connection.

former is usually connected "Delta" wherever possible.

The connection in Fig. 66 is used for three-phase rotary converters as shown in Fig. 68. If the rotary has six rings connected to six points, $A$, $B$, $C$, $A'$, $B'$, $C'$, as shown in Fig. 69 (this is called a

six-phase rotary), the same transformer will do, but each secondary is left independent. Fig. 69 will explain this connection. Another connection for a six-phase rotary, called the double "Delta" con-

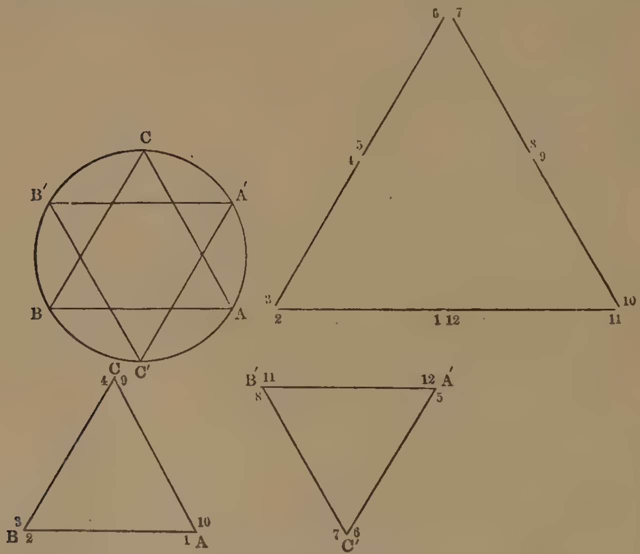


FIG. 70. Double Δ Connection for Six-Phase Rotary Converter.

nection, is shown in Fig. 70. This connection is more complicated than that shown in Fig. 69, and is less economical of copper because the rotary voltage is less.

**Two-phase to Three-phase Connections.**—For transmission a three-phase system is generally used

because it is most economical of copper. For distribution a two-phase current is, in some respects, simpler, and is, in some cases, preferred. Hence, the

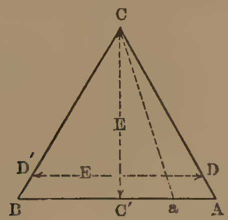


FIG. 71. Two-Phase to Three-Phase Connection.

necessity for transformation from three-phase to two-phase, or the reverse.

If we connect the secondaries of a bank of three-phase transformers "Delta," as shown in Fig. 71,

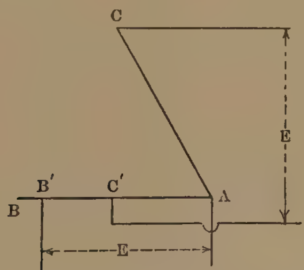


FIG. 72. Two-Phase to Three-Phase Connection.

pressures of different magnitude and phase may be obtained by tapping off at different points. For example, if we connect to $C$ and $a$ we get a pressure

of magnitude $Ca$ and phase $Ca$. If we connect to $C$ and $C'$ we get a pressure at right angles to $AB$ but smaller, $CC'$ being equal to $\frac{AB\sqrt{3}}{2}$. We may get a pressure equal to $CC'$ in magnitude and at right angles to it by connecting to two points $DD'$ such that the length $DD' = CC'$. This gives us a three-phase to two-phase transformation, or vice versa,

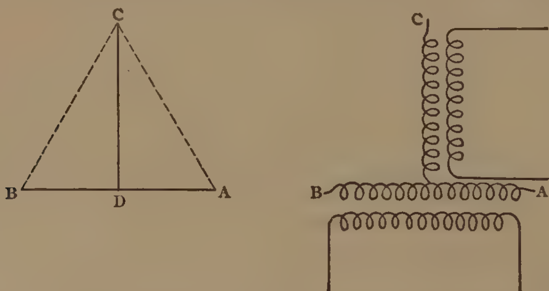


FIG. 73. Scott System of Two-Phase to Three-Phase Transformation.

with three transformers. Evidently, in a three-phase "Delta" connection, one transformer, say $BC$, may be left out as shown in Fig. 72. Two equal pressures may be obtained as shown in this figure. This gives, however, an unsymmetrical connection, the wire $AC'$ carrying the current of both sides of the two-phase system, unless a separate winding is put on. Another method of obtaining two equal pressures at right angles from a three-phase system is shown in Fig. 74.

Perhaps the best connection for transformation

from three-phase to two-phase, or from two-phase to three-phase, is that due to Mr. C. F. Scott and shown

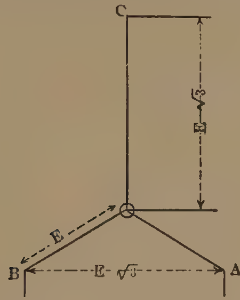


FIG. 74. Two-Phase to Three-Phase Connection.

in Fig. 73. The coil $C$ contains 86% $\left(=\frac{\sqrt{3}}{2}\right)$ as many turns as the coil $AB$, in order to give equal voltages across $AB$, $BC$, and $CA$.

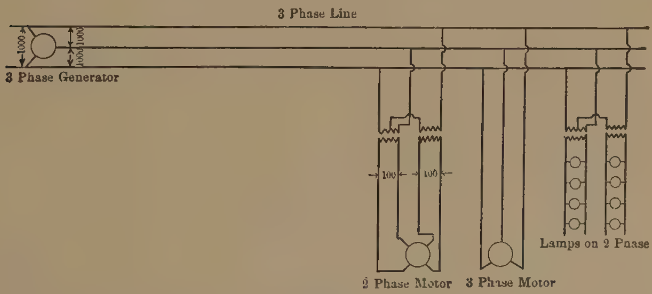


FIG. 75. Three-Phase Generation, Two and Three-Phase Distribution.

95. Fig. 75 shows a three-phase generator and Scott transformers for transforming to two-phase.

The diagram in Fig. 76 shows different methods of grouping three-phase transformers for polyphase transformation.

In the $T$ connection, one of the transformers is wound for $\frac{\sqrt{3}}{2}$ times the voltage of the other, and connected with one of its ends to the centre of the other.

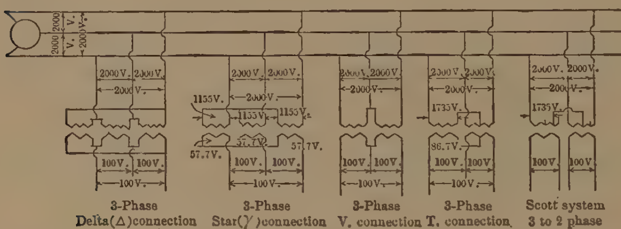


FIG. 76. Methods of Grouping Single-Phase Transformers for Poly-phase Transformations.

In a paper by W. L. R. Emmet, before the A. I. E. E., November 22, 1901, the three-phase system is strongly advocated in place of the single-phase or two-phase. The only difficulty in operating a three-phase system for a local distribution is to so divide the load that there will be little unbalancing of the phases. Mr. Emmet claims that this can generally be done.

Two systems of low tension distribution may be used: the three-phase, four-wire system, and the

single-phase, three-wire system. The three-phase, four-wire system is shown in Fig. 77, and the single-phase, three-wire in Fig. 78. The first method is generally the more advantageous as there is less

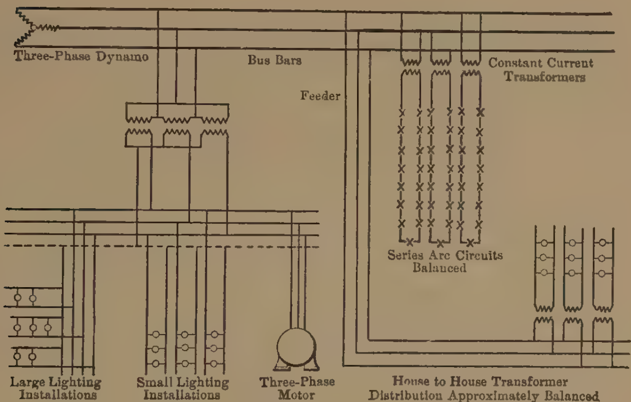


FIG. 77. Four-Wire, Three-Phase, Low Tension System.

unbalancing. There would be about 115 volts between neutral and line wire and 200 volts between the line wires. 200-volt motors may then be connected directly between the line wires.

Another system of distribution, which is coming into use, is shown in Fig. 79. The *Y* connected generator gives about 2,300 volts between neutral and outside wire and 4,000 volts between terminals. This system, which is clearly shown in the figure, has

a number of advantages over the other two shown in Figs. 77 and 78.

Mr. Emmet suggests that the following rules should always be kept in view when distributing systems are being arranged: 1st. Reduce the num-

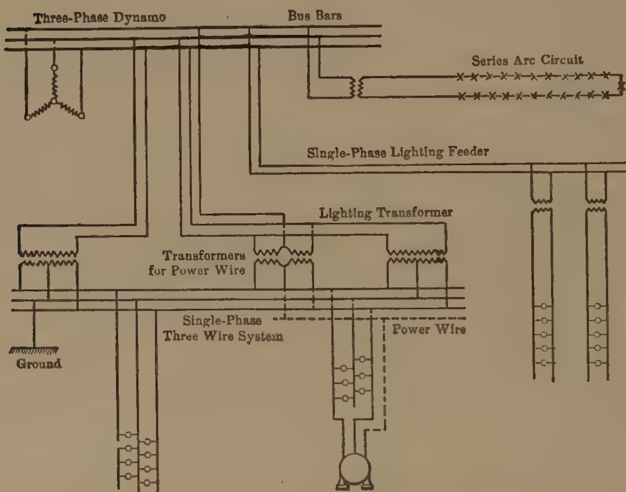


FIG. 78. Three-Phase Generator with Single Phase, Three-Wire, Low Tension System.

ber of circuits to be regulated to a minimum. 2d. So arrange the system that variations not controllable from the main station are made as small as possible. 3d. Reduce the amount of total drop on all circuits so that frequent changes will not be

necessary. 4th. Inter-connect the load in large groups, so that any variable load unit will be small

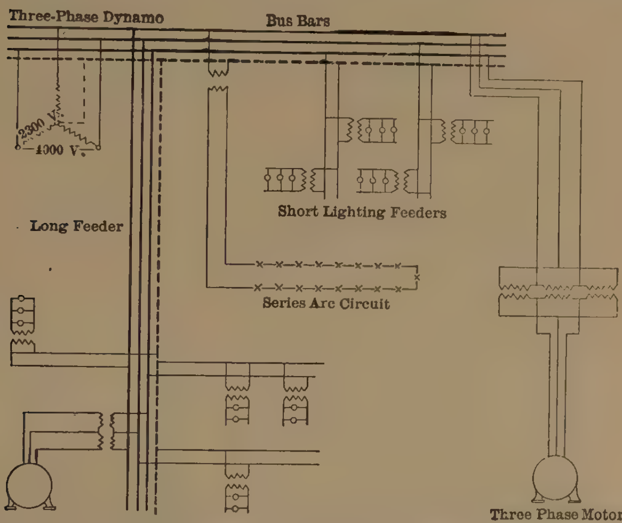


FIG. 79. High Tension, Four-Wire, Three-Phase System.

in comparison to the whole of which it forms a part. Figs. 77, 78 and 79 are from Mr. Emmet's paper.

## CHAPTER X.

### COMMERCIAL TYPES OF TRANSFORMERS.

I have attempted to collect such available material regarding commercial types of transformers as would

FIGS. 80 and 81. Converse Transformers.

give the reader some idea of the type, general appearance and constructional details of transformers

now on the market. All manufacturers were given equal opportunity to furnish material, but a few did not respond to requests for information in time.


FIG. 82. Water Cooled Converse Station Transformer, 333 K. W. for 80,000 volts.

## THE CONVERSE TRANSFORMER COMPANY, PITTSBURG, PA.

This company makes shell type transformers. Figs. 80 and 81 show the external appearance of the 1 to 50 K. W. sizes. Fig. 82 shows a 333 K. W.

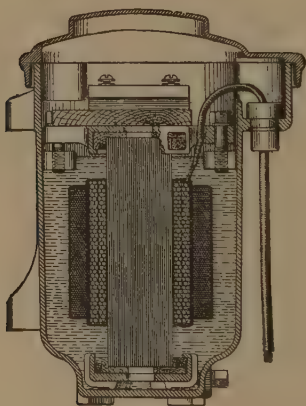

FIG. 83. General Electric Type H. Transformer, Section Through Case, Coils and Core.

transformer for an 80,000 volt, three-phase transmission system. This is claimed to be the highest voltage transformer for commercial work constructed.

FIG. 84. Parts of Type H. General Electric Transformers.

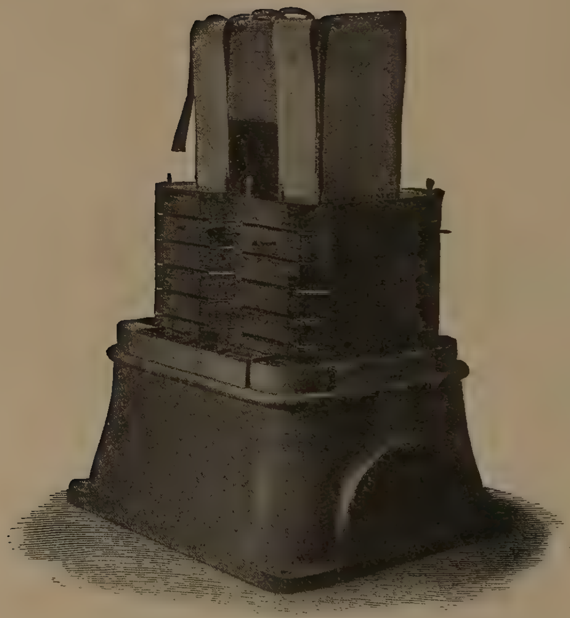

FIG. 85. Large General Electric Shell Type Transformer in Process of Construction.

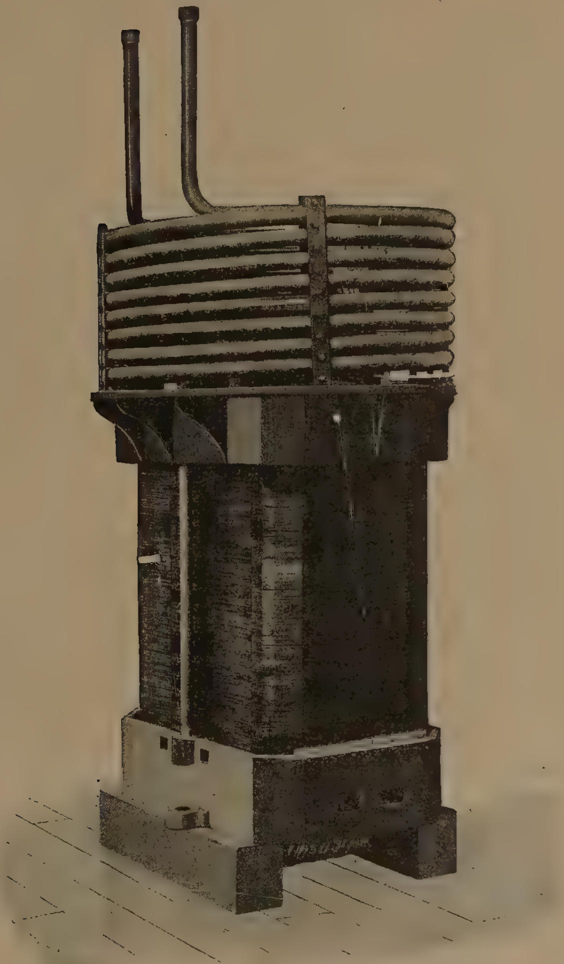

FIG. 86. 800 K. W. General Electric Water Cooled Large Transformer Without Case.

## GENERAL ELECTRIC COMPANY, SCHENECTADY, N. Y.

For lighting purposes, this company builds a core type transformer. As shown in Fig. 83, the transformer is provided with a cast iron case and the case is filled with oil. Fig. 84 shows the parts of a core type transformer.

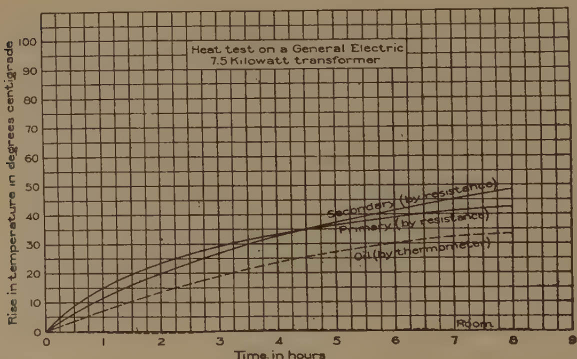


FIG. 87. Heat Test on General Electric 7.5 K. W. Transformer.

For transformers of large capacity the shell type of construction is used. Fig. 85 shows a shell type transformer in process of construction. These transformers are made for cooling by air blast, Fig. 85, or water-cooled as in Fig. 86.

Fig. 87 shows a heat test of a 7.5 K. W. core type transformer.

Fig. 88 shows the method of installing transformers for th ee-wire distribution.

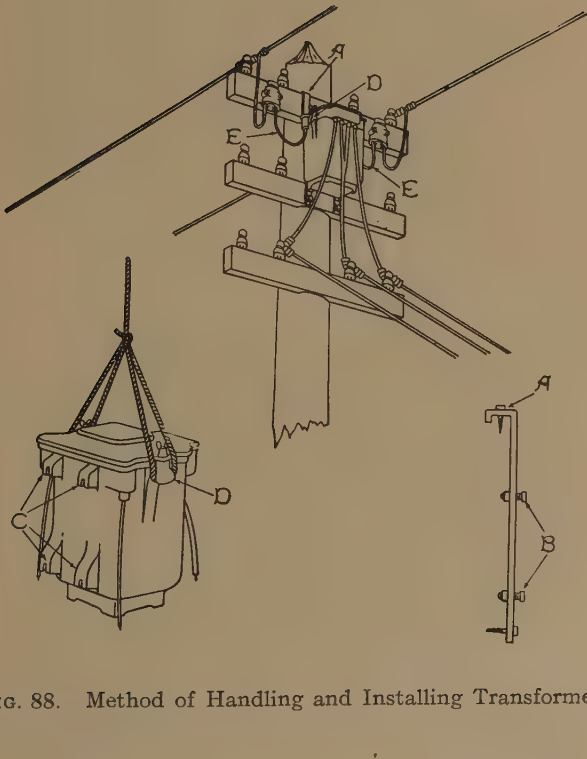


FIG. 88. Method of Handling and Installing Transformer.

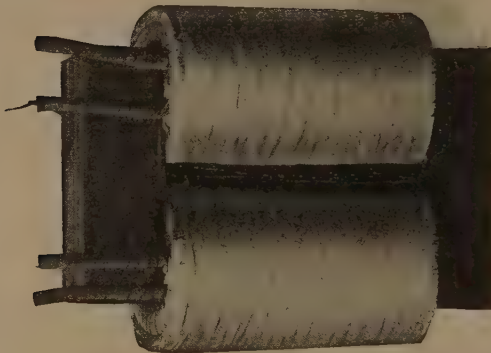

FIG. 90.

FIG. 89. 7.5 K. W. Lakon Transformer.

## THE LAKON COMPANY, ELKHART, IND.

A core type transformer is manufactured by this company. The general appearance, externally and internally, is shown by Figs. 89 to 91.

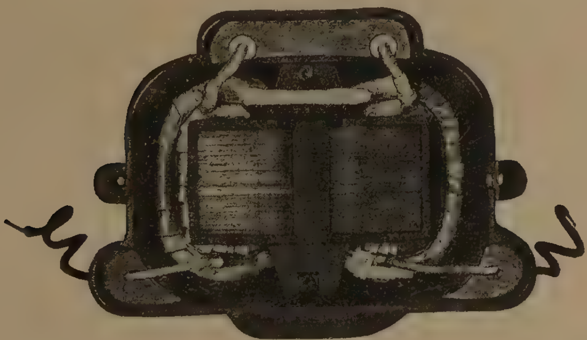

FIG. 91. Interior of Lakon Transformer.

MOLONEY ELECTRIC CO., ST. LOUIS, MO.

This company also makes core type transformers. The general appearance and construction are shown in Figs. 92 to 93.

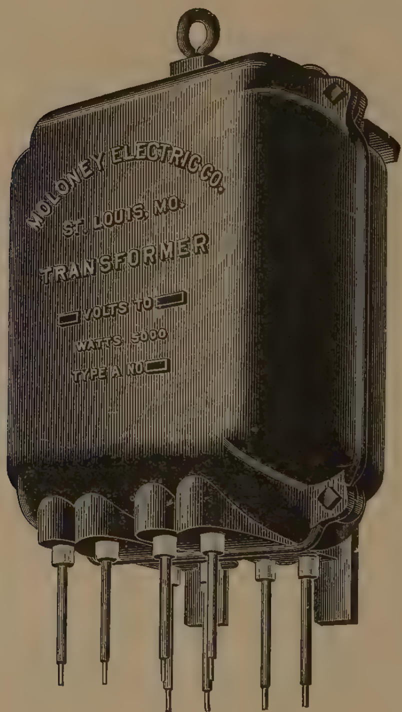


FIG. 92. Type A 5 K. W. Moloney Transformer. Front View Showing Lead Wires and General Appearance.

FIG. 93. Type A, Moloney Transformer, Showing Ventilating Slots and Air Circulation in and around the Transformer Coils.

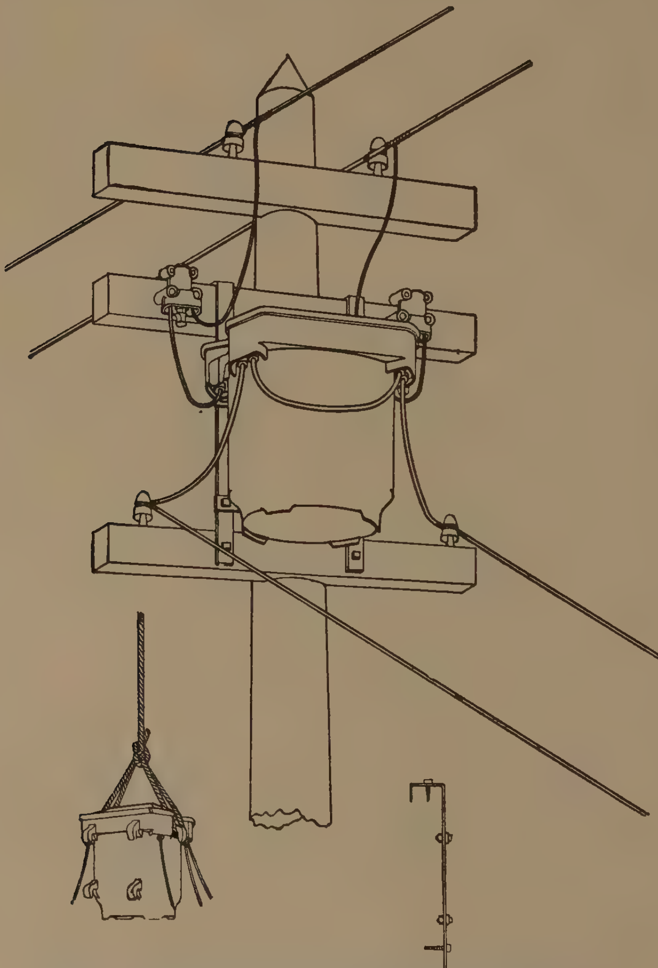

FIG. 94. Method of Instaling Packard Transformer.

NEW YORK AND OHIO CO., WARREN, OHIO.

"Packard" transformers are manufactured by this company. This is a core type transformer. The general appearance and method of installing are shown in Fig. 94. The arrangement of coils and core is shown in Fig. 95.

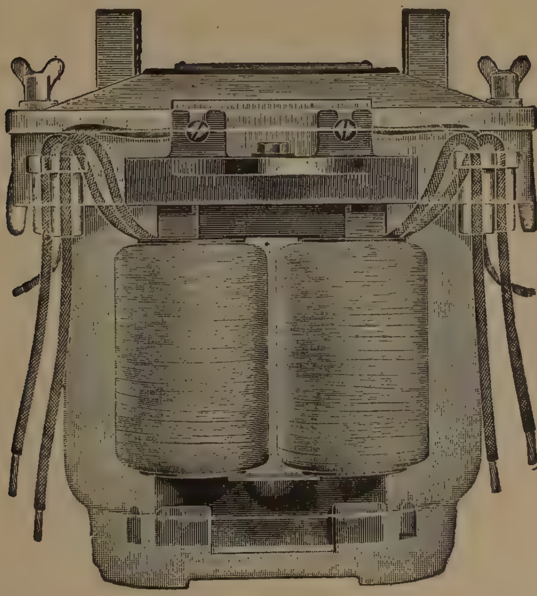

FIG. 95. Arrangement of Core and Coils in Case of Packard Transformer.

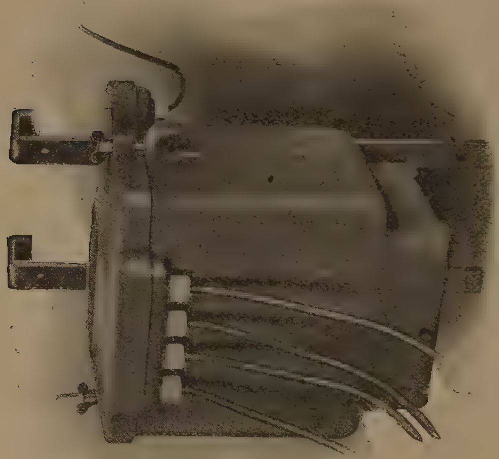

FIG. 97. S. K. C. Type A. O. Transformer.

FIG. 96. Coils and Core of Stanley A. O. Transformer.

## STANLEY ELECTRIC CO., PITTSFIELD, MASS.

This company was one of the first to manufacture transformers on a commercial scale in this country. Both shell and core types are constructed. For its lighting transformers the company uses the shell type as shown in Fig. 96. Figs. 96 to 98 show the general design. The transformer case is filled with oil.

This company has made a great many transformers for high voltage transmission systems.*

FIG. 98. Method of Separately Winding and Insulating Coils of 10 K. W. and 4 K. W. Type A. O. Stanley Transformer.

*For illustrated description of the high voltage transformers in the power house of the Bay Counties Power Co., at Colgate, California, by Mr. Edw. Heitmann and Wm. Currie, see the *Journal of Electricity, Power and Gas*, for April, 1902.

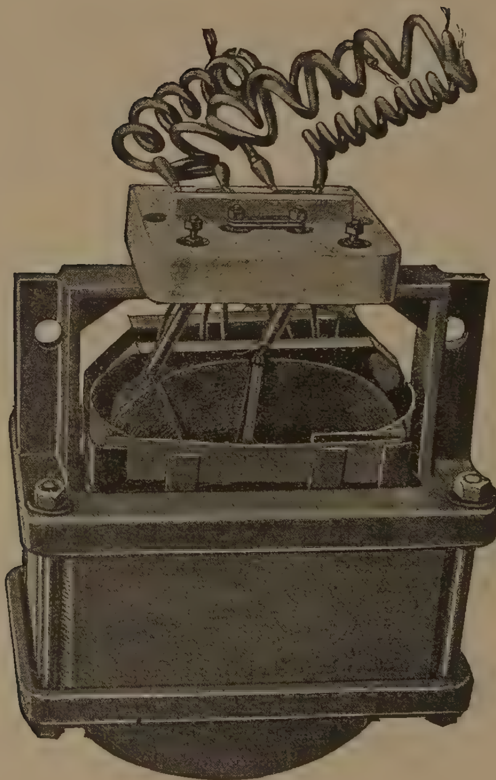

FIG. 99. Two K. W., O. D. Westinghouse Transformer without Case.

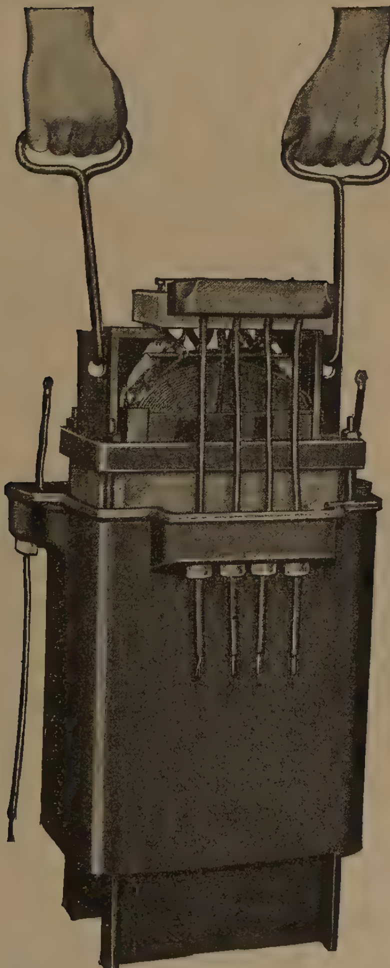

FIG. 100. Lifting O. D. Westinghouse Transformer from Case.

FIG. 101. Parts of O. D. Westinghouse Transformer.

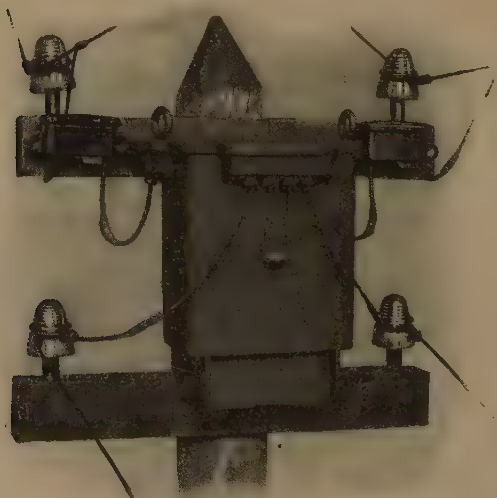

FIG. 102. Method of Installing Westinghouse Transformer.

## WESTINGHOUSE ELECTRIC AND MFG. CO.
## PITTSBURG, PA.

This company makes a shell type transformer exclusively. Its design is very neat and compact.

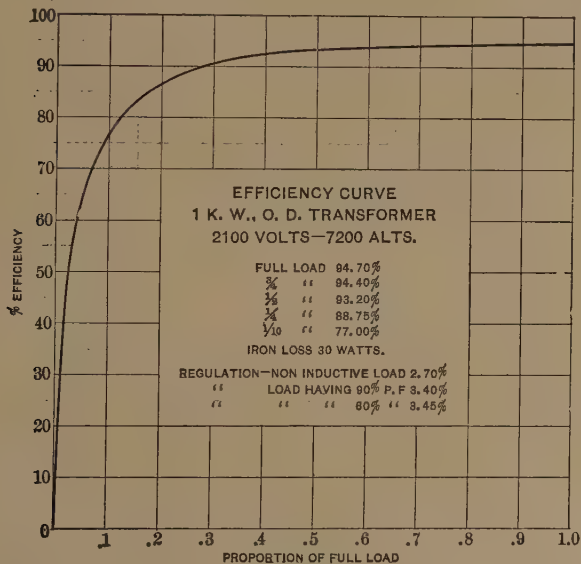


FIG. 103. Efficiency Curve, Westinghouse Transformer.

The case is oil filled. Figs. 99 to 102 will give a good idea of the design and construction.

Figs. 103 and 104 give the electrical characteristics of these transformers. The standard O. D. transformers, for 60 cycles, of from 1 to 5 K. W., may be operated at any frequency from 40 to 1,333,

and the standard 5 to 50 K. W. may be operated at 25 to 30 cycles. Small transformers are made self-cooling. Large transformers are made self-cooling or water-cooled.

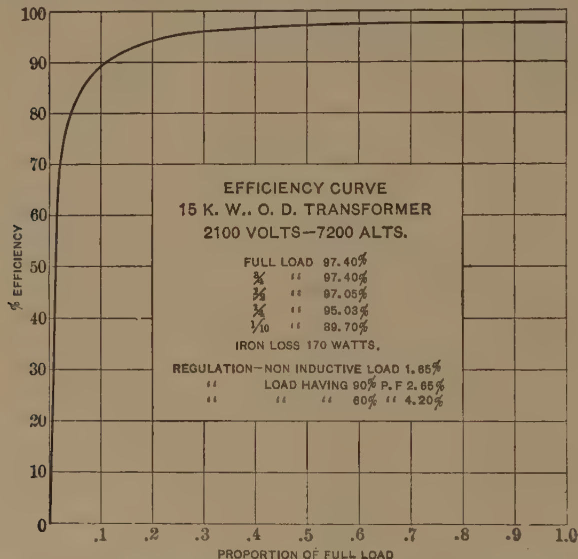


FIG. 104. Efficiency Curve, Westinghouse Transformer.

Fig. 105 gives the results of a heat test, showing clearly the advantage of the oil-filled transformer.

Fig. 106 shows the time required to raise the temperature 50° C, with different loads on the Wagner transformer. A large percentage overload may be carried for a short time without injuring the transformer.

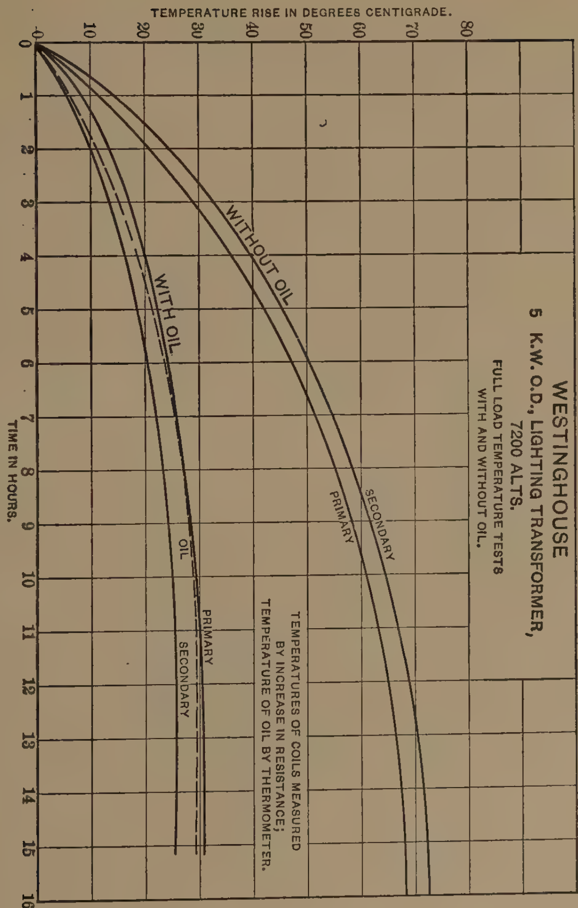


FIG. 105. · Temperature Test, Westinghouse Transformer.

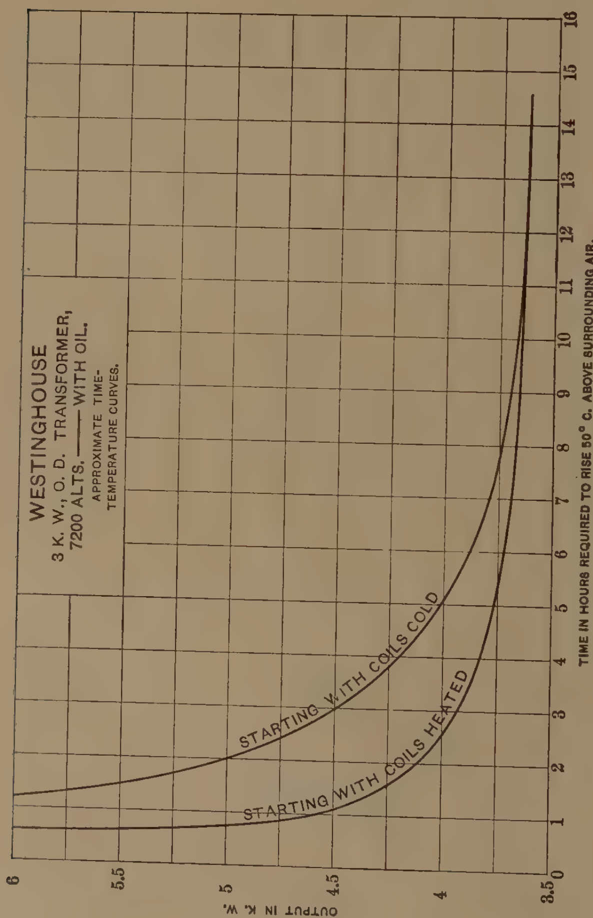


FIG. 106. Time Temperature Curve, Westinghouse Transformer.

## ALLGEMEINE ELEKTRICITATS GESELLSCHAFT.

This company uses the core type, both for single-phase and three-phase transformers. Its method of constructing three-phase transformers is shown in Fig. 107.

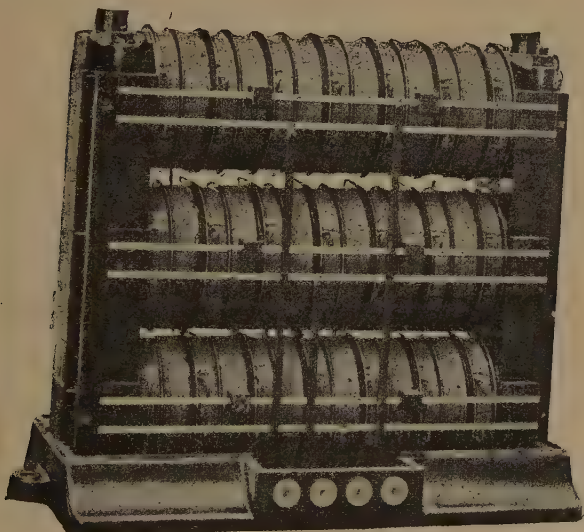

FIG 107. Three-Phase Transformer.

## BROWN, BOVERIE & CO.

For single-phase work this company makes a transformer similar to that made by the Maschinen

Fabrik Oerlikon, and shown in Fig. 122. For polyphase work the firm uses single-phase transformers. For transformer stations the firm uses the tower

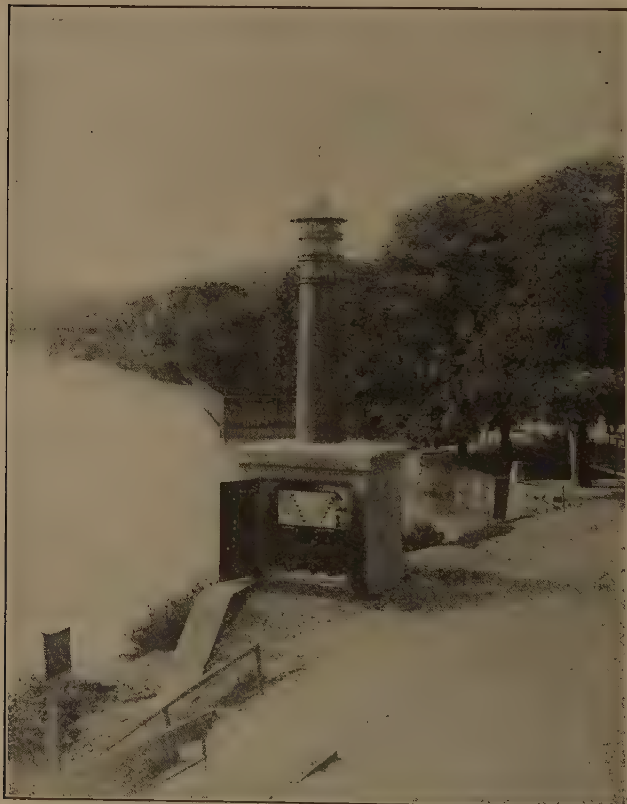

FIG. 108. View of Transformer Tower; Secondary Side Open.

construction shown in Fig. 108, which shows a tower in the Olten-Aarburg system. The primary side of the tower is shown in Fig. 109.

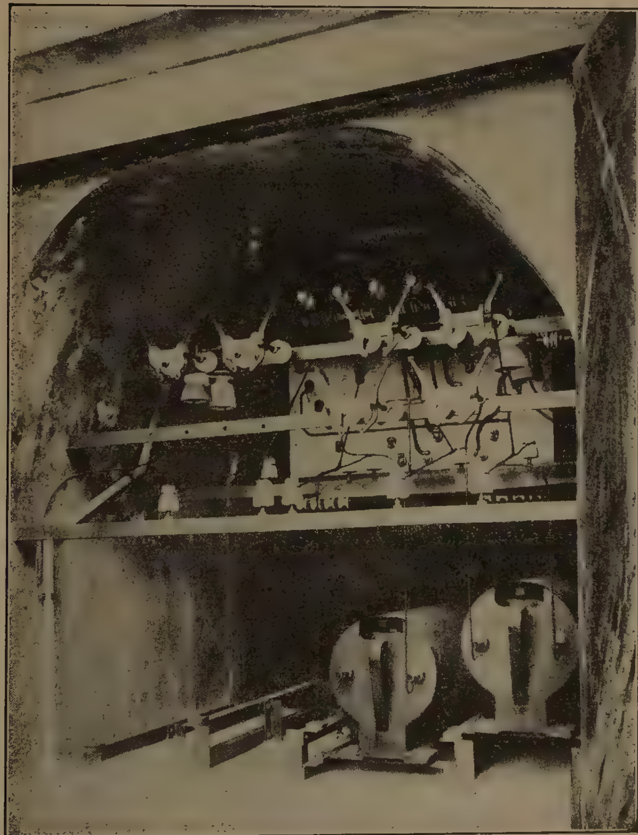

FIG. 109. Interior of Transformer Tower; Primary Side.

## BRUSH ELECTRICAL ENGINEERING CO.

This company makes shell type transformers. A special transformer constructed to comply with the requirements of the Board of Trade for use in underground sub-stations is shown in Fig. 110.

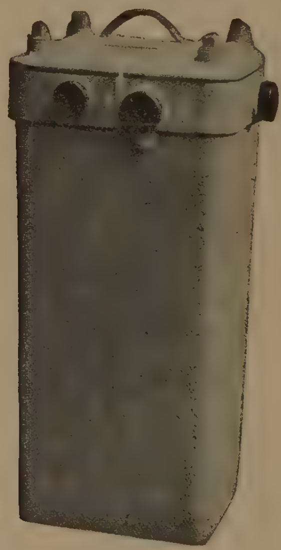

FIG. 110. Transformer for Underground Sub-stations.

## ELECTRICITATS-AKTIEN-GESELLSCHAFT.

For single-phase work this company uses the core type of construction. The company's three-phase transformer is shown in Fig. 111. As shown, the length of magnetic circuit for each phase is the same.

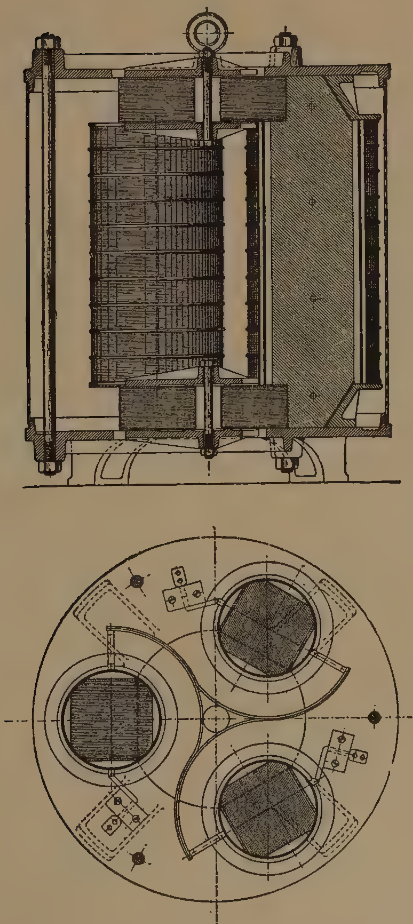

FIG. 111. Three-Phase Transformer.

## FERRANTI TRANSFORMERS.

Figs. 112 to 114 show a Ferranti standard transformer in position for lowering into a sub-station tank, and also the transformer in tank complete. The transformer is wholly enclosed in a water-tight tank, the only joint being made by machined surfaces bolted together and packed with suitable material.

FIG. 112.

Special attention is given to the shielding of all high tension terminals and fittings. The circular chamber at the top contains a Cardew earthing device, the high tension and low tension terminals which are designed for single, concentric or two-core cables, provision being made for sealing the ends of paper cables. This chamber can also be fitted, if desired, with high tension and low tension fuses. The cable glands and lid joint are water-tight, and the latter is especially designed for easy and quick removal.

These tanks are made in three sizes to take 20, 30, or 50 K. W. transformers; they will take, if necessary, the intermediate sizes of transformers.

FIG. 113.

FIG. 114.

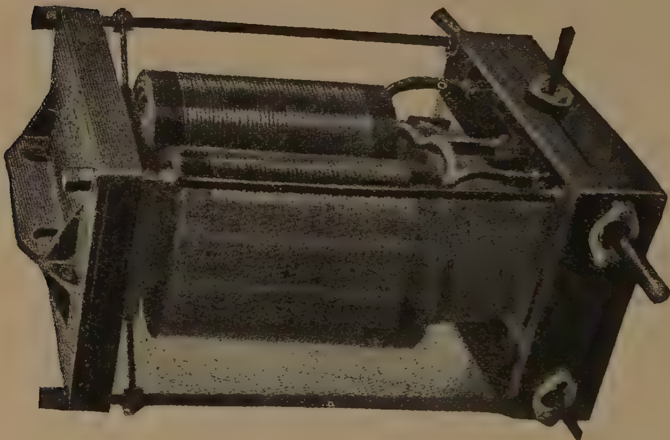

FIG. 116.

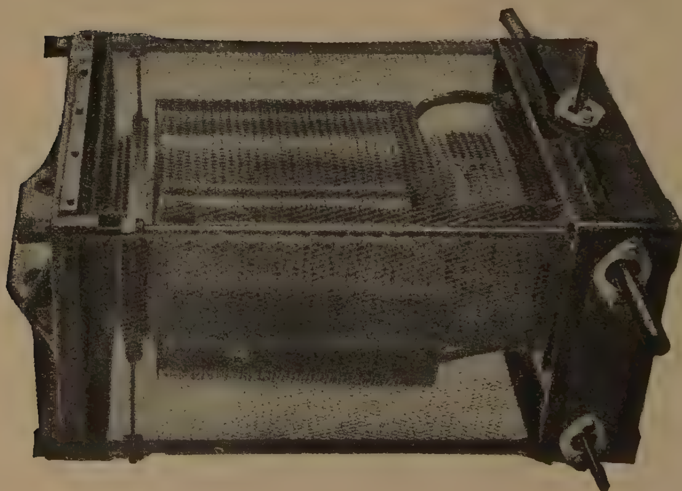

FIG. 115.

The illustrations, Figs. 115 and 116, show one of the standard types of Ferranti transformer and ventilated cover. This construction is specially applicable for use in places which are perfectly dry, such as main generating stations or well constructed substations.

The transformer proper is clamped together by two solid plates at the top and bottom. The tcp plate is furnished with eye-holes for lifting purposes. The bottom plate is in cast iron and contains receptacles for receiving cable terminals, etc. The cables enter through glands (shown in illustrations), and it is possible, after these have been connected up to the terminals, to run the whole recess in with a preservative compound, thus affording efficient means for sealing the ends of the cables and ensuring against break-downs on these connections.

(1.) Higher efficiency at heavy loads owing to better ventilation.

(2.) Larger output if the temperature rise is the same as in its counterpart in an air-tight tank.

(3.) Less initial cost, due to cheaper construction of the containing cases.

The following are standard sizes: 5, 10, 15, 20, 30, 40, 50, K. W. for periodicities of or about 50 and 100 cycles per second.

## GANZ & Co.

This company, like most of the European companies, makes single-phase and three-phase trans-

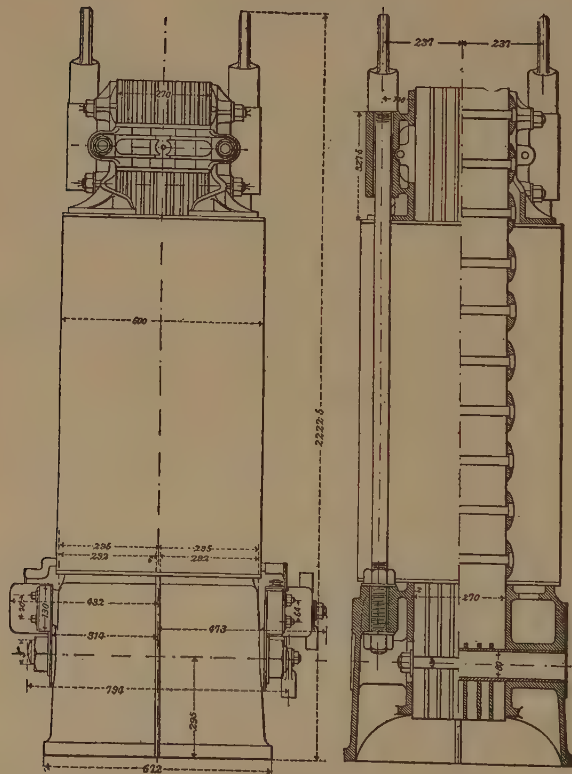


FIG. 117.

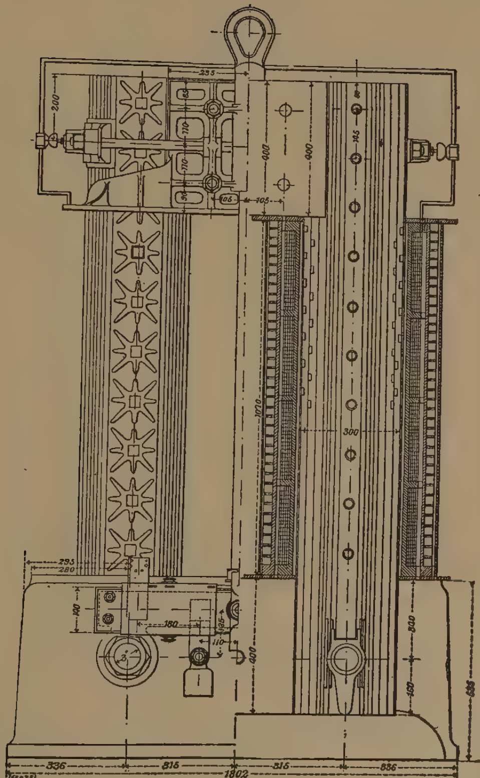
235
200
400
400
105
1070
300
295
280
140
180
400
336
816
816
836
1802

FIG. 118.

formers. Shell-type and core-type are used for single-phase. The three-phase transformers have three laminated columns of square cross-section, disposed in one plane, the ends of the columns being united by two laminated end pieces. On these columns the primary and secondary are laid alternately, and are fixed by means of projecting plates of the end pieces. For holding these plates together, and

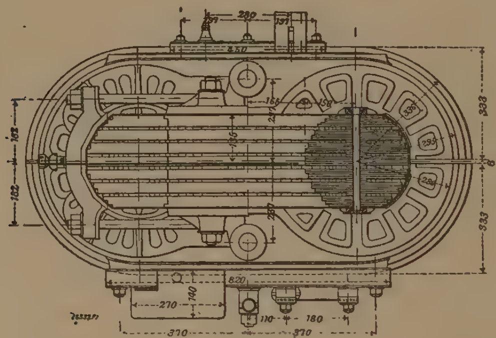

FIG. 119.

in order to secure the transformer to the ring-shaped base, there are employed four bolts passing through cast iron pipes. The polyphase transformers now made by Messrs. Ganz & Co. are of this type. The large sizes are somewhat different, but only in the construction of the base.

A 300 K. W. single-phase transformer for a current of 25 periods, 5,000 volts primary and 430 volts secondary, is shown in Figs. 117 to 119. The core of this transformer is composed of two circular col-

umns of iron plates and has connecting-parts, of two horizontal bundles of plates; the iron core, therefore, forms a simple magnet circuit. The whole transformer rests on a cast iron base, composed of two parts, enclosing the under connections of the iron core. The two parts of the base are held by bolts, while the cast iron piece holding the upper part together is secured by means of two forged iron rods, so that the whole iron construction is strong and rigid. Directly on the iron core are the primary coils, and surrounding them are the secondary coils, in such a way that the air may circulate freely between them. For the same purpose the iron core of the transformer, in the direction of the plates, is provided with air openings, securing good ventilation. The secondary coils are made of copper bars of .98 inch square. In this transformer, when loaded for a long time with 300 K. W. and cooled by a 1 H. P. centrifugal fan, the temperature will not rise more than 25° C above the surrounding air. This transformer has a weight of 4 tons and at full load an efficiency of 98.3 per cent.

## MASCHINEN FABRIK OERLIKON.

For three-phase transformers this company's type of construction is shown in Fig. 120, which is for 300 K. W.

A special type of air cooled, single-phase, core type transformer is shown in Figs. 121a and 121b. The winding is aluminum.

Fig. 122 shows the single transformer made by this company The core is built up as shown, of plates of different width so as to approximate a cylindrical shape, and these are held together by gun metal, side-pieces and bolts.

FIG. 120. Three-Phase Transformer, 300 K. W.

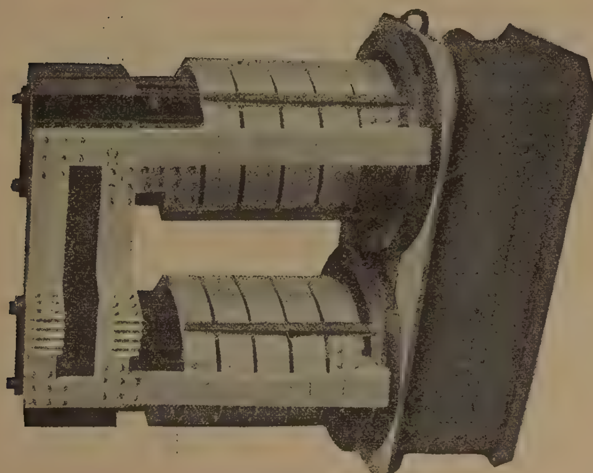

FIG. 121a. Showing Secondary Winding.

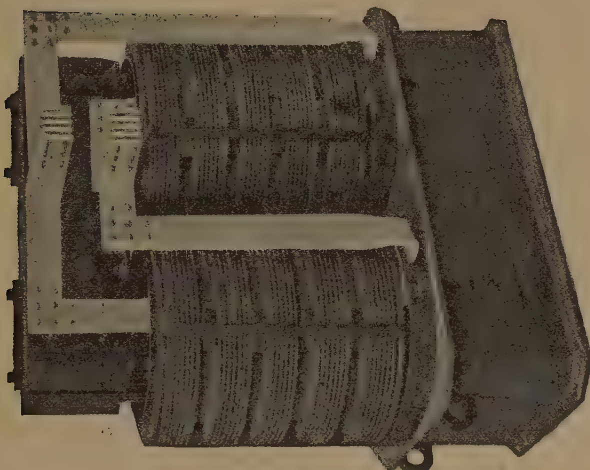

FIG. 121b. Showing Both Windings.

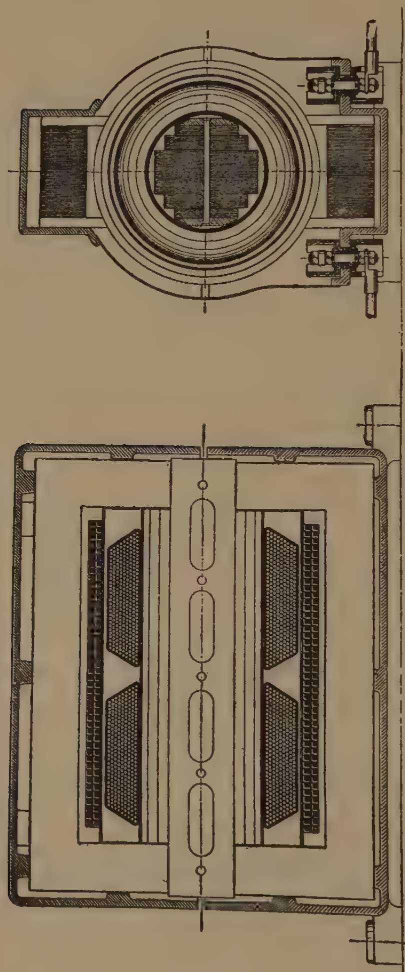

FIG. 122. Single-Phase Transformer.

# APPENDIX.

EXTRACTS FROM RULES AND REQUIREMENTS OF THE NATIONAL BOARD OF FIRE UNDERWRITERS.

## Construction and Tests.

(*a*) Must not be placed in any but metallic or other non-combustible cases.

(*b*) Must be constructed to comply with the following tests:

(1.) Shall be run for eight consecutive hours at full load in watts under conditions of service, and at the end of that time the rise in temperature, as measured by the increase of resistance of the primary coil, shall not exceed 135 degrees Fahrenheit.

(2.) The insulation of transformers when heated shall withstand continuously for five minutes a difference of potential of 10,000 volts (alternating) between primary and secondary coils and between the primary coils and core, and a no-load "run" at double voltage for thirty minutes.

## Location.

(1.) Must not be placed inside of any building, excepting central stations, unless by special permission of the Inspection Department having jurisdiction.

(2.) Must not be attached to the outside walls of

buildings, unless separated therefrom by substantial supports.

(3.) In central or sub-stations the transformers must be so placed that smoke from the burning out of the coils or the boiling over of the oil (where oil filled cases are used) could do no harm.

(4.) Must be located at a point as near as possible to that at which the primary wires enter the building.

(5.) Must be placed in an enclosure constructed of fire-resisting material; the inclosure to be used only for this purpose, and to be kept securely locked and access to the same allowed only to responsible persons.

(6.) Must be effectually insulated from the ground and the inclosure in which they are placed must be practically air-tight, except that it shall be thoroughly ventilated to the outdoor air, if possible, through a chimney or flue. There should be at least six inches air-space on all sides of the transformer.

## Grounding Low-Potential Circuits.

(1.) The ground connection for Central Stations, transformer sub-stations, and banks of transformers must be made through metal plates buried in coke below permanent moisture level, and connection should also be made to all available underground piping systems, including the lead sheath of underground cables.

(2.) For individual transformers and building services the ground connection may be made as in

Section *f*, or may be made to water or other piping systems running into the buildings. This connection may be made by carrying the ground wire into the cellar and connecting on the street side of meters, main cocks, etc., but connection must never be made to any lead pipes which form part of gas services.

In connecting ground wires to piping systems, where possible, the wires should be soldered into one or more brass plugs and the plugs forcibly screwed into a pipe-fitting, or where the pipes are cast iron, into a hole tapped to the pipe itself. For large stations, where connecting to underground pipes with bell and spigot joints, it is well to connect to several lengths, as the pipe joints may be of rather high resistance. Where such plugs can not be used, the surface of the pipe may be filed or scraped bright, the wire wound around it, and a strong clamp put over the wire and firmly bolted together.

Where ground plates are used, a No. 16 copper plate, about three by six feet in size, with about two feet of crushed coke or charcoal, about pea size, both under and over it, would make a ground of sufficient capacity for a moderate size station, and would probably answer for the ordinary sub-station or bank of transformers. For a large Central Station considerable more area might be necessary, depending upon the other underground connections available. The ground wire should be riveted to such a plate in a number of places, and soldered for its whole length. Perhaps even better than a copper plate is a cast iron plate with projecting forks, the idea of

the fork being to distribute the connection to the ground over a fairly broad area, and to give a large surface contact. The ground wire can probably best be connected to such a cast iron plate by brass plugs screwed into the plate to which the wire is soldered. In all cases, the joint between the plate and the ground wire should be thoroughly protected against corrosion by suitable painting with waterproof paint or some equivalent.

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